Collaborative Spectrum Sensing from Sparse Observations Using Matrix Completion for Cognitive Radio Networks

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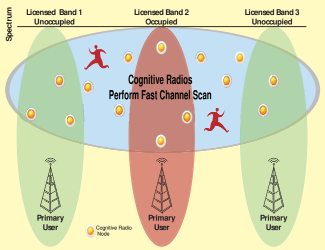
Contribution

We apply recent matrix completion techniques to greatly reduce the sensing information needed for spectrum hole detection in cognitive radio networks. The key idea is to equip each Cognitive Radio (CR) node with a frequency selective filter, through which the powers of multiple wide-band channels are combined linearly to form a small number of channel sensing report. The collaborative spectrum sensing problem in cognitive radio networks is further formulated as two sub-problems: matrix completion sub-problem and joint-sparsity reconstruction sub-problem.

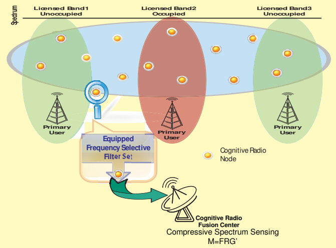
Spectrum Sensing in Cognitive Radio Networks

- Cognitive radio has been known as a novel paradigm for *improving radio spectrum utilization*;
- *Spectrum sensing* is a key enabler for cognitive radio;
- Single CR node has only limited local observation to the whole spectrum due to various constraints, *collaborations* among CR nodes (CSS) are important for acquiring the complete spectrum information;
- Spectrum scann performed by each CR is *time consuming*;
- *Power limitation and channel fading* limited the available channel sensing information, *Missing and erroneous reports* due to random transmission loss are inevitable;

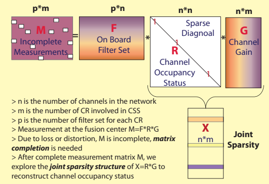
System Model & Algorithm



a) Limitation of Traditional CSS



b) Proposed Compressive Spectrum Sensing



c) Signal Model

System model:

$$\mathbf{M}_{p\times m} = \mathbf{F}_{p\times n}\mathbf{R}_{n\times n}(\mathbf{G}_{m\times n})^{\top}. \quad (1)$$

Incomplete measurement matrix:

$$\mathbf{M}^{E}_{ij} = \begin{cases} \mathbf{M}_{ij}, & \text{if } (i,j)\in \mathbf{E}, \\ 0, & \text{otherwise.} \end{cases} \quad (2)$$

Matrix completion:

$$\min_{\mathbf{M}\in\mathbb{R}^{p\times n}} \tau\|\mathbf{M}\|_* + \frac{1}{2}\sum_{(i,j)\in\mathbf{E}} \left|\mathbf{M}_{i,j} - \mathbf{M}^{E}_{i,j}\right|^2, \quad (3)$$

Joint sparsity recovery:

Algorithm 1 Joint Detection Algorithm

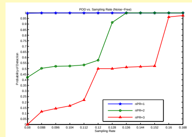
$T \leftarrow \{1,\dots,n\}$
repeat
  Independent recovery:
  $\mathbf{X} \leftarrow 0$
  $X_{\cdot,j} \leftarrow \min\{\sum_{i\in T} X_{i,j} : A_j X_{\cdot,j} = b_j, X_{\cdot,j} \geq 0\}$ for every CR j with enough measurements (In presence of measurement noise, $A_j X_{\cdot,j} = b_j$ is replaced by $\|A_j X_{\cdot,j} - b_j\| \leq \sigma$)
  Channels detection:
  select trusted $X_{\cdot,j}$ and detect used channels from the selections
  Update of T**:**
  Update T according to detected channels and $\mathbf{X}$
until the tail of $\mathbf{X}$ is small enough
Report $\mathbf{X}$, and $\mathbf{R}$ by thresholding $\mathbf{X}$

Simulations

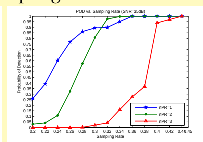
According to FCC and Defense Advance Research Projects Agency (DARPA) reports data, we chose to test the proposed algorithm for CSS with such settings:

- A set of 100 channels;
- 20 CR nodes collaboratively detecting the occupied channels;
- Each CR has a set of 50 frequency selective filters;
- At certain times, the number of active primary users varies from 1 to 3;
- Performance is tested for noisefree and SNR=35dB.

POD vs. Sampling Rate



a) Noise-Free



b) SNR=35dB

We define sampling rate as

$$\frac{No.\ received\ measurements\ at\ the\ fusion\ center}{No.\ channels \times No.\ CRs},$$

Performance was given in terms of probability of detection (POD) according to the definition in information-theory.

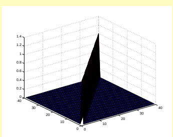
$$POD = No.\ Hit/(No.\ Hit + No.\ Miss);$$

The Art of Matrix Completion

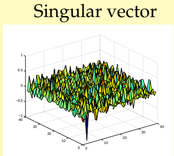
Latested development in mathematics claims that if a matrix satisfies the following condition, we can fullfill it with confidence from a small number of its **uniformly random** revealed entries.

- **Low Rank**: Only a small number of none-zero singular values;
- **Incoherent Property**: Singular vectors well spread across all corodinate.

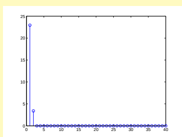
Singular vector



a) spiky



b) Spread across all coordinates



c) Sparse singular value

Resemble the l1 norm minimization for finding the sparse solution to compressive sensing problem. Low rank matrix can be reconstructed through nuclear norm minimization follow a two steps algorithm:

- Rank prediction;
- Nuclear norm minimization.

Conclusions

- We innovatively equip each CR with a frequency selective filter set, so that each CR can get a linear combination of the entire channel occupancy information throughout the network simultaneously. This will greatly *reduce the time for spectrum sensing*.
- We model the CSS problem as a *matrix completion problem* in which partial observations of a matrix enable its faithful reconstruction.
- We solve the matrix completion problem using the recent algorithm FPCA and estimate channel availability through exploring the *joint sparsity structure*.
- In the noiseless cases, exact detection was obtained with no more than 8% of the complete sensing information, whilst as the number of primary user increases, to achieve a detection rate of 95.55%, the required information percentage was merely 16.8%. In the noisy cases (SNR: 35dB), less than 15% samples enabled exact detection of small numbers of primary users.

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