Collaborative Spectrum Sensing from Sparse Observations Using Matrix Completion for Cognitive Radio Networks

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\textbf{Contribution}

We apply recent matrix completion techniques to greatly reduce the sensing information needed for spectrum hole detection in cognitive radio networks. The key idea is to equip each Cognitive Radio (CR) node with a frequency selective filter, through which the powers of multiple wide-band channels are combined linearly to form a small number of channel sensing report. The collaborative spectrum sensing problem in cognitive radio networks is further formulated as two subproblems: matrix completion subproblem and joint-sparsity reconstruction subproblem.

\textbf{System Model & Algorithm}

\begin{equation}
M_{p,n} = F_{p,n}R_{p,n}(G_{m,n})^T.
\end{equation}

\begin{equation}
M_{ij}^E = \begin{cases} M_{ij}, & \text{if } (i,j) \in E \text{,} \\ 0, & \text{otherwise.} \end{cases}
\end{equation}

\begin{equation}
\min_{M \in \mathbb{R}^{n \times m}} \| M \|_F + \frac{1}{2} \sum_{(i,j) \in E} |M_{ij} - M_{ij}^E|^2.
\end{equation}

\textbf{Joint sparsity recovery:}

\textbf{Simulations}

According to FCC and Defense Advance Research Projects Agency (DARPA) reports data, we chose to test the proposed algorithm for CSS with such settings:

- A set of 100 channels;
- 20 CR nodes collaboratively detecting the occupied channels;
- Each CR has a set of 50 frequency selective filters;
- At certain times, the number of active primary users varies from 1 to 3;
- Performance is tested for noise-free and SNR=35dB.

We define sampling rate as

\textbf{POD vs. Sampling Rate}

\textbf{POD = No. Hit/(No. Hit + No. Miss)}

At certain times, the number of active primary users varies from 1 to 3.

\textbf{The Art of Matrix Completion}

Latested development in mathematics claims that if a matrix satisfies the following condition, we can fulfill it with confidence from a small number of its uniformly random revealed entries.

- **Low Rank:** Only a small number of non-zero singular values;
- **Incoherent Property:** Singular vectors well spread across all coordinates.

Resemble the II norm minimization for finding the sparse solution to compressive sensing problem. Low rank matrix can be reconstructed through nuclear norm minimization follow a two steps algorithm:

- Rank prediction;
- Nuclear norm minimization.

\textbf{Conclusions}

- We innovatively equip each CR with a frequency selective filter set, so that each CR can get a linear combination of the entire channel occupancy information through the network simultaneously. This will greatly reduce the time for spectrum sensing.
- We model the CSS problem as a matrix completion problem in which partial observations of a matrix enable its faithful reconstruction.
- We solve the matrix completion problem using the recent algorithm FPCA and estimate channel availability through exploring the joint sparsity structure.
- In the noiseless cases, exact detection was obtained with no more than 8% of the complete sensing information, whilst as the number of primary user increases, to achieve a detection rate of 95.55%, the required information percentage was merely 16.8%. In the noisy cases (SNR = 35dB), less than 15% samples enabled exact detection of small numbers of primary users.

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\textbf{Spectrum Sensing in Cognitive Radio Networks}

- Cognitive radio has been known as a novel paradigm for improving radio spectrum utilization;
- Spectrum sensing is a key enabler for cognitive radio;
- Single CR node has only limited local observation to the whole spectrum due to various constraints, collaborative among CR nodes (CSS) are important for acquiring the complete spectrum information;
- Spectrum scan performed by each CR is time consuming;
- Power limitation and channel fading limited the available channel sensing information. Missing and erroneous reports due to random transmission loss are inevitable.