Sampling Rate Reduction for 60 GHz UWB Communication Using Compressive Sensing

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***Abstract*— 60 GHz ultra wide-band (UWB) communication is an emerging technology for high speed short range communications. However, the requirement of high-speed sampling increases the cost of receiver circuitry such as analog-to-digital converter (ADC). In this paper, we propose to use a compressive sensing framework to achieve a significant reduction of sampling rate. The basic idea is based on the observation that the received signals are sparse in the time domain due to the limited multipath effects at 60 GHz wireless transmission. According to the theory of compressive sensing, by carefully designing the sensing scheme, sub-Nyquist rate sampling of the sparse signal still enables exact recovery with very high probability. We discuss an implementation for a low-speed A/D converter for 60 GHz UWB received signal. Moreover, we analyze the bit error rate (BER) performance for BPSK modulation under RAKE reception. Simulation results show that in the single antenna pair system model, sampling rate can be reduced to 2.2% with 0.3dB loss of BER performance if the input sparsity is less than 1%. Consequently, the implementation cost of ADC is significantly reduced.**

I. Introduction

The dogma of signal processing maintains that a signal must be sampled at a frequency, at least twice its bandwidth in order to be represented without error. However, in practice, we often compress the data soon after sensing, trading off signal representation complexity (bits) for some error (consider JPEG image compression in digital cameras, for example). Clearly, this is wasteful of valuable sensing/sampling resources. Over the past few years, a new theory of "compressive sensing" [1–3] has begun to emerge, according to which the signal is sampled (and simultaneously compressed) at a greatly reduced rate if the input signal has the sparse property. Very recently, compressive sensing has been used in several wireless communication and networking applications [5], [6], [10].

60 GHz communication is an emerging technology for high-speed short-range wireless communications. Recently, there has been an increasing interest in providing broadband communications in the 60 GHz unlicensed band for wireless personal area networks (WPANs). A wide available spectrum, e.g. 57-64 GHz for North American services, surrounding the 60 GHz operating frequency has the ability to support high-rate broadband wireless systems such as voice, data, and full-motion real-time video for home entertainment applications [7]. Due to special propagation characteristics at this frequency range multipath effects still exist for indoor applications and they must be precisely characterized for diversity transmission purposes. Ultra wide-band (UWB) signals are much less sensitive to these effects [7]. Hence, to take the benefits of both 60 GHz and UWB communications we up-covert the UWB signal by 60 GHz carrier. For this system, so-called 60G UWB, the requirement of high-speed sampling significantly increases the cost of RF circuitry such as the analog-to-digital converter (ADC) at the receiver.

In this paper, we propose to utilize the concept of compressive sensing to achieve a significant reduction of sampling rate. The basic idea is based on the observation that the received signals are sparse in the time domain due to the limited multipath effects at 60 GHz wireless transmission. The low information rate enables us to employ much lower sampling rate while still be able to reconstruct the signal with a low probability of error. We describe an implementation for a low-speed A/D converter for 60 GHz UWB received signal. We also analyze the bit error rate (BER) performance for BPSK under RAKE reception. Simulation results show that, the sampling rate can be reduced to only 2.2% for the signal with sparsity of 1%, and the BER performance loss is only 0.3 dB.

The rest of this paper is organized as follows: Section II presents the low speed A/D conversion system model for 60GHz UWB communication system. Section III explains prototype implementation of the proposed ADC using compressive sensing framework, and investigate the performance of RAKE BPSK receiver. Simulation results are presented and analyzed in Section IV. Finally, conclusions are drawn in Section V.

II. System Model

We consider a system with the transmission signal $s(t)$ and the received signal written as

$$x(t) = \sum_{k=1}^{K} h_k s(t - \tau_k) + \epsilon(t) \qquad (1)$$

where K is the number of multipaths, $\epsilon(t)$ is the thermal noise. h_k is the k^{th} multipath response, and τ_k is its delay. There are two distinct characteristics for 60G UWB transmission. First, because the narrow impulse in the time domain, the

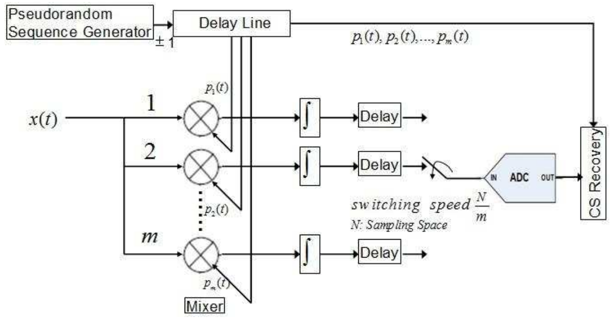

Fig. 1. Compressive sensing-based ADC.

receiver needs high sampling rate. Second, due to the line-of-sight transmission and high attenuation due to reflection, there are few multipaths (with sufficiently high power). As a result, the number of multipaths K is much smaller than the sampling space. This signal sparsity motivates and enables us to utilize a compressive sensing framework.

In the 60G UWB system, the transmitted pulse is an up-converted UWB pulse at 60 GHz carrier. An accurate ray-tracing model is used to simulate the multipath propagation channel. It was confirmed that at 60 GHz frequency, the channel characterization results obtained by accurate ray-tracing method are quite reliable and can be verified by measured data [8], [9]. Since in the indoor applications at 60 GHz, the high penetration loss of the construction material isolates adjacent rooms and significantly limits the received interference, the ray-tracing simulation needs to be applied only for the objects inside the room. In this simulation, we consider a typical furnished office room as a propagation environment (see Fig. 2). In this model, a patch antenna transmitter is mounted a few cms off the ceiling at the center and beam facing down. The patch antenna receiver is located right on the center of the meeting table and facing upward. We consider the antenna pattern as a part of the channel. The transmitter has 10 dBm transmit power. The transmitter/receiver antenna gain is set to 7 dB. All the materials in the room set for 60 GHz frequency in terms of complex permittivity.

The 60G UWB pulses received by the receiver antenna are then amplified using a wide-band low noise amplifier and processed via a compressive sensing receiver described in the next section.

III. Compressive Sensing-Based ADC

In this section, we first propose an implementation for a low-speed A/D converter for 60 GHz UWB received signal. Then we formulate the compressive sensing problem, and investigate how to use the *a priori* source information for signal recovery at the receiver based on the compressive sensing model. Finally, the BER performance of BPSK is analyzed under RAKE reception.

A. Proposed ADC

Viewed as composition of a discrete, finite number of weighted continuous basis or dictionary components (described in equation (2)), multipath received signal is sparse in

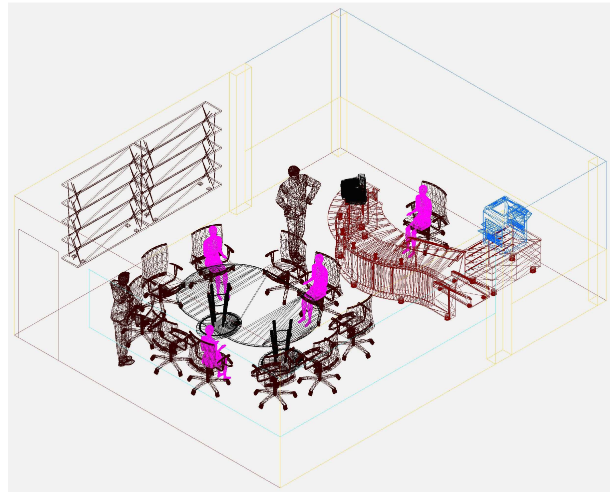

Fig. 2. Simulated environment in ray-tracing.

time domain. Due to the line-of-sight transmission and high attenuation due to reflection, there are few multi-paths with sufficient high power.

$$x(t) = \sum_{n=1}^{N} \alpha_n \delta_n(t) \tag{2}$$

In Fig. 3, we show a multipath power delay profile obtained using the accurate ray-tracing modeling. When the noise floor is assumed to be -100 dBm, as can be observed from the figure, only a few multipaths exist. The average delay for indoor environment is 100 ns, and the sampling rate for 60G UWB can be 100 ps. As a result, in a time frame, we have a sampling space of $N = 1000$, within which only a small number of significant multipath coefficients exist. Obviously $\mathbf{X}$ is sparse.

The proposed system, shown in Fig. 1 is composed of three major parts: 1) The mixer, in which, the multipath received signal is modulated by a set of pseudo-random (PN) Bernoulli sequence, and the alternation between values in one sequence is at or faster than the Nyquist rate of the input signal; 2) The integrator, which integrate the mixer output throughout each time frame; 3) Multiple copies of mixer and integrator chain, with total number of copies needed M decided by the signal sparsity level. Usually, $3 \sim 5$ times of the number of significant component in a time frame is enough for exact signal recovery. After these three parts, a low speed ADC is implemented, its input simply switches among the M copies of mixer and integrator chain, and thus the A/D conversion speed decreases to M per time frame.

We introduce a delay line after the PN Bernoulli sequence generator. Each $P_i(t)$ shown in Fig. 1 is a row vector with Bernoulli i.i.d. entries, and by delaying the output of the PN Bernoulli sequence generator, $P_2(t), \cdots, P_m(t)$ are time shifted copies of $P_1(t)$. These time shifted copies used for signal modulation not only simplify the PN generation part, but also help save the storage space in the CS recovery part. Besides, we introduce sample and hold at the integrator output which can hold the integrator output at the end of each time

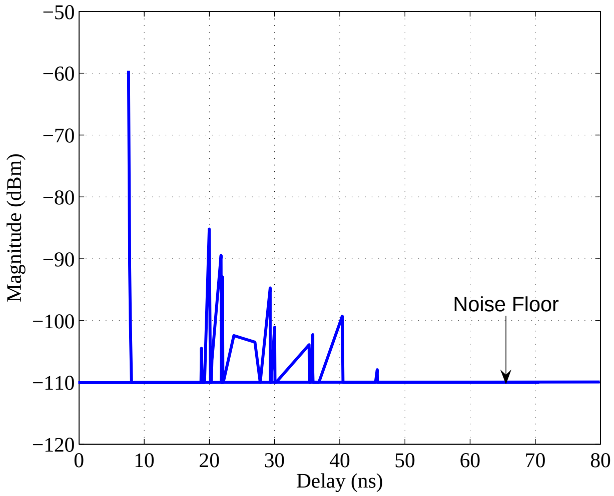

Fig. 3. A simulated power delay profile for 60 GHz UWB transmission.

frame for the ADC switch to arrive. In such a way, only one ADC is needed for M chains of mixer and integrator.

Output of the ADC is a matrix Y, which is given by

$$\mathbf{Y}_{M\times 1} = \mathbf{P}_{M\times N}\mathbf{X}_{N\times 1}. \tag{3}$$

Each column vector with length M holds the quantized integrator output within a time frame from M copies of the compressive sensing chain. Each row vector is the quantized integrator output throughout the time. At the end of each time frame, one column will be taken out, together with the PN matrix $\mathbf{P}$ for compressive sensing signal reconstruction.

B. Formulation of the Compressive Sensing Problem

The problem is to obtain the K significant components of $\mathbf{X}$ using the limited number (M) of measurements. The first question is whether or not the information of K-sparse signal is damaged by the reduction of dimension from $\mathbf{X} \in \mathbb{R}^N$ down to $\mathbf{Y} \in \mathbb{R}^M$. In general, if $\mathbf{X}$ is not sparse enough, as long as $M < N$, the signal is damaged since there are fewer equations than unknowns. On the other hand, for the K-sparse signal, $\mathbf{Y}$ is just a linear combination of K columns of $\mathbf{P}$. A necessary and sufficient condition to ensure that this $M \times K$ system can be compressed and reconstructed is stated as the following property:

Definition 1: Restricted Isometry Property (RIP) [1–3]: For any vector $\mathbf{V}$ sharing the same K nonzero entries as $\mathbf{X}$, if

$$1 - \epsilon \le \frac{\|\mathbf{PV}\|^2}{\|\mathbf{V}\|^2} \le 1 + \epsilon \tag{4}$$

for some $\epsilon > 0$, then the matrix $\mathbf{P}$ preserves the information of the K-sparse signal. A sufficient condition for stable inverse in practice is that $\mathbf{P}$ satisfies (1) for an arbitrary $3K$-sparse vector $\mathbf{V}$.

In [1–3], it was proved that if $\mathbf{P}$ is a random ± 1 entry matrix, then the K-sparse signal is compressible with high probability if $K \le cM\log(N/M)$, where c is a constant. It was later proved that Toeplitz-structured matrices also satisfy the RIP condition. For our problem, we design $\mathbf{P}$ in such a way that, the first row of $\mathbf{P}$ is a PN Bernoulli sequence, and the rows after are time shifted copies of row one, therefore it is a Toeplitz matrix with Bernoulli i.i.d entries which satisfies the RIP condition.

From RIP, we know that under a certain condition the K signals are still preserved in M dimensions. The next question is how to develop a reconstruction algorithm to recover $\mathbf{X}$ from the measured $\mathbf{Y}$. Since $M < N$ there are infinite number of $\hat{\mathbf{X}}$ satisfying $\mathbf{Y} = \mathbf{P}\hat{\mathbf{X}}$. All solutions lie on the $N-M$ dimension hyperplane $\mathcal{H} := \mathcal{N}(\mathbf{P}) + \mathbf{X}$ which corresponds to the null space $\mathcal{N}(\mathbf{P})$ translated to $\mathbf{X}$. Therefore, the problem is to find the sparse reconstructed signal $\hat{\mathbf{X}}$ in the translated null space as:

$$\hat{\mathbf{X}} = \arg\min_{\mathbf{Y}=\mathbf{P}\hat{\mathbf{X}}} |\hat{\mathbf{X}}|_1 \tag{5}$$

where $|\cdot|_1$ is norm one. It was shown in [15] that for norm two, there might be many solutions, and for norm zero, the complexity is NP hard [1–3]. The problem in (5) is a convex optimization problem, which can be reduced to a linear program, which is called the l_1-magic in the literature. The complexity is $O(N^3)$. However, a much simpler algorithms such as the simplex algorithm [11], [12] can be easily employed.

C. Bayesian Detection

To directly solve the problem in (5) using $l-1$ magic, the performance might not be good. This is because it does not utilize the prior information about the source. In this subsection, we adopt the Bayesian compressive sensing [13], [14], which is fully probabilistic. It introduces a set of hyper-parameters which is viewed as *a priori* probability of the signal, and the most probable values are iteratively estimated from the received data.

1) Model Specification: In (1), the noise in the system is composed of propagation loss with zero mean and variance σ^2. The probability density function can be approximated as a Gaussian distribution as

$$p(\epsilon) = \prod_{i=1}^{M} \mathcal{N}(\epsilon_i|0,\sigma^2). \tag{6}$$

Due to the assumption of independence of Y_n, the likelihood of the complete data set can be written as

$$p(\mathbf{Y}|\mathbf{P},\sigma^2) = (2\pi\sigma^2)^{-M/2}\exp\left(-\frac{1}{2\sigma^2}\|\mathbf{Y}-\mathbf{PX}\|^2\right). \tag{7}$$

By adopting a Bayesian perspective, we constrain the parameters by defining an explicit *a priori* probability distribution over them. Here, we assume a zero-mean Gaussian *a priori* distribution over the signal $\mathbf{X}$[1]

$$p(\mathbf{X}|\alpha) = \prod_{n=1}^{N} \mathcal{N}(X_i|0,\alpha_i^{-1}) \tag{8}$$

where α is a vector of N independent hyper-parameters.

[1]Even though $\mathbf{X}$ is not Gaussian distributed, simulation results show that the performance indeed improves.

Given α, the posterior parameter distribution conditioned over the signal is given by combining the likelihood and prior with Bayes' rule:

$$p(\mathbf{X}|\mathbf{Y},\alpha,\sigma^2) = \frac{p(\mathbf{Y}|\mathbf{X},\sigma^2)p(\mathbf{X}|\alpha)}{p(\mathbf{Y}|\alpha,\sigma^2)} \quad (9)$$

which is a Gaussian distribution $\mathcal{N}(\mu,\mathbf{\Sigma})$ with covariance and mean of $\mathbf{\Sigma} = (\mathbf{A} + \sigma^{-2}\mathbf{P}^T\mathbf{P})^{-1}$ and $\mu = \sigma^{-2}\mathbf{\Sigma}\mathbf{P}^T\mathbf{Y}$, respectively, where $\mathbf{A} = diag(\alpha_1,\ldots,\alpha_N)$.

2) Marginal Likelihood Maximization: A most-probable point estimate α_{MP} may be found via a type-II maximum likelihood procedure [18]. The sparse Bayesian model is formulated as the local maximization with respect to α of the marginal likelihood, or equivalently its logarithm:

$$\begin{aligned} L(\alpha) &= \log p(\mathbf{Y}|\alpha,\sigma^2) \quad (10)\\ &= \log \int_{-\infty}^{\infty} p(\mathbf{Y}|\mathbf{X},\sigma^2)p(\mathbf{X}|\alpha)dX \\ &= -\frac{1}{2}\left(M\log 2\pi + \log|\mathbf{C}| + \mathbf{Y}^T\mathbf{C}^{-1}\mathbf{Y}\right) \end{aligned}$$

with $\mathbf{C} = \sigma^2 + \mathbf{I} + \mathbf{P}\mathbf{A}^{-1}\mathbf{P}^T$.

A point estimate μ_{MP} for the parameters is then obtained by evaluating μ with $\alpha = \alpha_{MP}$, giving a posterior mean approximator $\mathbf{PX} = \mathbf{P}\mu_{MP}$. However, marginal likelihoods are generally difficult to compute, i.e., values of α and σ^2 which maximize $L(\alpha)$ cannot be obtained in closed form. Thus, we need to re-estimate them iteratively.

For updating α, following the approach in [18], we differentiate (10), and then equate it to 0. After rearranging, we have $\alpha_i^{new} = \frac{\gamma^i}{\mu_i^2}$, where μ_i is the i^{th} posterior mean signal μ, and γ_i is defined as $\gamma_i = 1 - \alpha_i N_{ii}$ with N_{ii} being the i^{th} diagonal element of the posterior signal covariance $\mathbf{\Sigma}$ computed with current α and σ^2 values. Each γ_i can be treated as a measure of how well-determined its corresponding parameter X_i is by the data. For the variance σ^2, differentiation leads to re-estimate:

$$\sigma^2_{new} = \frac{\|\mathbf{Y} - \mathbf{P}\mu\|^2}{M - \sum_i \gamma_i}. \quad (11)$$

We repeat calculation of α and σ^2 with iteratively updating μ and $\mathbf{\Sigma}$ until certain convergence criteria have been reached. This procedure leads to the maximization of marginal likelihood.

Then at the convergence of α estimation procedure, we make predictions based on the posterior distribution over the signal, conditioned on the maximizing values α_{MP} and σ^2_{MP}. In other words, by doing this, we could pick up those entries in the projection matrix $\mathbf{P}$ which after projection preserves the information of the signal in $\mathbf{Y}$. By utilizing the corresponding elements in the measurements $\mathbf{Y}$ and projection matrix $\mathbf{P}$, we could reconstruct our signal with a better probability.

D. BER Analysis for RAKE Receiver

A RAKE receiver is a radio receiver designed to counter the effects of multipath fading, using several several correlators (fingers) each assigned to a different multipath component. Each finger independently decodes a single multipath component, and then the contribution of all fingers are combined in order to make the most use of the different transmission characteristics of each transmission path. Rake receivers are common in a wide variety devices such as mobile phones and wireless LAN equipments. In this subsection, we study the BER performance of BPSK under RAKE reception.

For traditional high data rate ADC, the received SNR under RAKE reception can be written as

$$\Gamma = \sum_{i\in\mathcal{N}} \frac{|X_i|^2}{N_0} \quad (12)$$

where $\mathcal{N}$ is the set of multipath components with sufficiently high power. For the proposed ADC, after reconstruction, we have $\hat{X}_i$. Therefore, the reconstructed SNR is given by

$$\hat{\Gamma} = \sum_{i\in\mathcal{N}'} \frac{|X_i\hat{X}_i|}{N_0} \quad (13)$$

where $\mathcal{N}'$ is the set of multipath components found by the reconstructed algorithm. If BPSK is employed, the BER is given by BER= $Q\left(\sqrt{2\hat{\Gamma}}\right)$.

IV. Simulation Results and Analysis

To verify the quality of our design, diverse types of simulations are carried out. We firstly simulate the data flow on the hardware, shown in Fig. 4. PN Bernoulli sequence we used to modulate multipath received signal is shown at the top, the modulated signal is shown in the middle, and compressive sensing received signal at the low speed ADC within a time frame is shown at the bottom. We also test the proposed Bayesian frame for CS signal recovery and one example of exact signal recovery is shown in Fig. 5.

In Fig. 6, we show the covariance between the original signal and the reconstructed signal by compressive sensing under different SNR. Figure shows that in the high SNR regime, the correlation is high and the sampling rate can be reduced to 1.6% without excessive performance loss when compared with higher sampling rate results. When the SNR is low, by increasing sampling rate we can still achieve very good performance. When the sampling rate is around 0.4%, the whole system collapses. Clearly, a tradeoff exists for M as a function of K and N.

In Fig. 7, we show the BER performance of BPSK under RAKE reception. For comparison, we show the BER performance of the proposed scheme. We can see that with a sampling rate of 4%, the proposed scheme almost has the optimal performance. When the sampling rate is reduced to 2.2%, in low SNR regime, the performance degrades. For sampling rate equal to 1.6%, the performance degrades even for high SNR. Again, when the sampling rate is 0.4%, the system collapses.

V. Conclusions

To lower the sampling rate while still achieving the performance of high-speed ADC, we have proposed a compressive sensing-based system utilizing one low-speed ADC. Based on the sparse property of the multipath received signal, a new

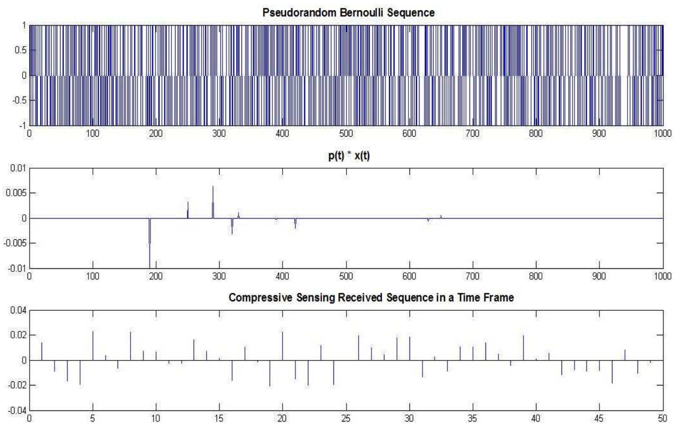

Fig. 4. Example of data flow on the hardware.

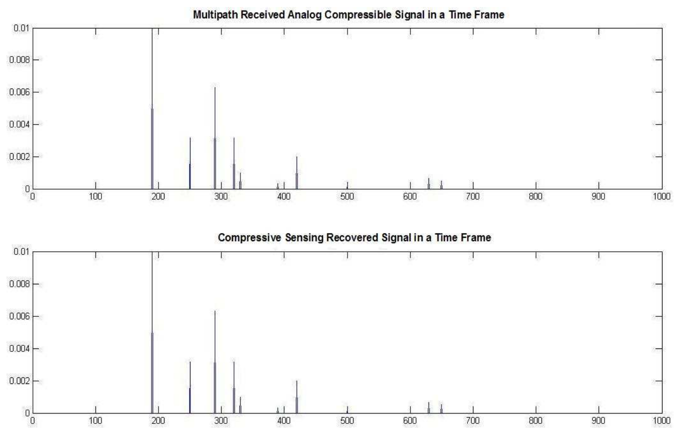

Fig. 5. Original signal vs. CS recovered signal.

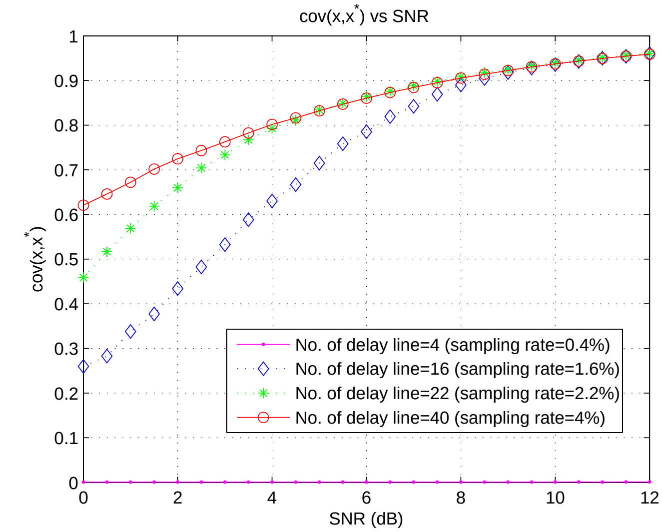

Fig. 6. Covariance between the original signal and reconstructed signal.

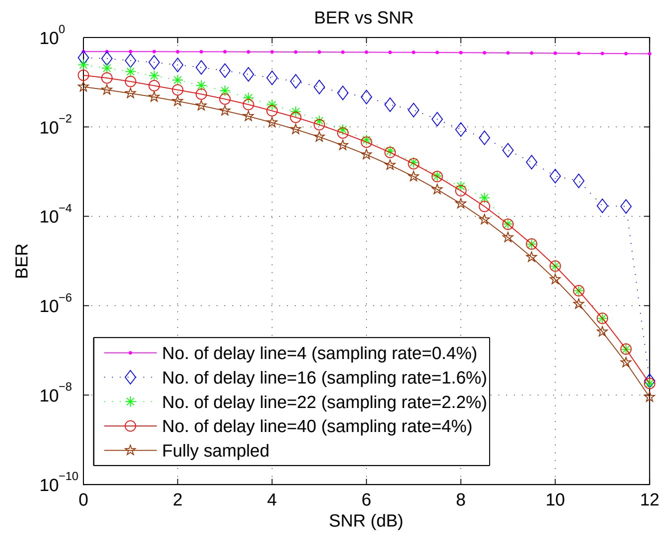

Fig. 7. BER performance under RAKE reception.

structure of low-speed sampling has been proposed and a compressive sensing problem has been formulated. To efficiently solve the problem, a Bayesian solution has been investigated. Simulation results have revealed that the BER of BPSK RAKE receiver has similar performance (0.3 dB loss) as the high speed one with the sampling rate as low as 2.2% in case the multipath received signal is sparse enough.

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