

# Pricing Game With Complete or Incomplete Information About Spectrum Inventories for Mobile Virtual Network Operators

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**Abstract**—In network virtualization, mobile virtual network operators (MVNOs) lease spectrum resources from mobile network operators (MNOs) and offer certain wireless services to end users. Each MVNO competes with others using optimal price and spectrum inventory so as to profit. However, the inventory of one MVNO is private information and maybe unknown to others, which makes pricing decisions difficult for MVNOs. In this work, we first study the pricing strategy given others' inventory information. We model the pricing decision problem using the non-cooperative game theory, and develop an optimal price setting algorithm based on an ordinal potential game. Then we put forward three cooperation strategies for MVNOs and analyze the impacts of the coalitions structure on pricing decision. For the situation where the inventory of one MVNO is unknown to others, we use the Bayesian coalition formation game to formulate the pricing decision problem and propose an optimal price setting algorithm based on the Minimum Mean-Square Error to resolve the conflicts resulting from the uncertainty. Next we define a Belief Pareto Order to characterize the preferences of MVNOs regarding the coalition structures. Then we devise a distributed coalition formation algorithm with the proposed belief Pareto order to achieve a Bayesian-Nash stable coalition structure that enables each MVNO to maximize its own revenue. Finally, simulation results demonstrate comprehensive performance evaluation of the proposed game model and provide a guidance on pricing strategies for MVNOs.

**Index Terms**—Mobile network operators, mobile virtual network operators, price competition, spectrum inventory, incomplete information, non-cooperative game, Bayesian coalition game.

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## I. INTRODUCTION

SINCE radio spectrum scarcity limits the number of mobile network operators (MNOs), mobile virtual network operators (MVNOs), enabled by wireless network virtualization, emerge as one comprehensive method for infrastructure sharing. MVNOs lease spectrum resources from MNOs and offer certain wireless services to end users (EUs) [1]. In recent years, the number of MVNOs is rapidly growing [2], it increased by 70 percent between June 2010 and June 2015, and reached 1017, as of June of 2015 [3]. To attract more EUs, one MVNO has to provide better service (e.g., higher quality of service (QoS) via more spectrum inventory or bundling their service with other products) or lower price (e.g., different pricing plans).

The interactions among MNOs, MVNOs and EUs can be modeled as a three-stage dynamic process [4]–[7], where MNOs and MVNOs make coordinately spectrum leasing decisions in Stage I, and both of them make pricing decisions in Stage II, and then EUs make subscribe decisions in Stage III. During Stage I, MVNOs obtain the spectrum inventories through the advance reservation, the on-demand requests, or the mixed model (see [8]–[11] and references therein). In advance reservation, each MVNO leases a fixed amount of spectrum resources from one MNO for a long time [12]. However, the uncertainty of EUs' demands may lead to the under-reservation or over-reservation. Thus, on-demand requests provide flexibility for MVNOs to make inventory decisions according to the observed demands in real time [13]. Nevertheless, there is no guarantee of sufficient spectrum supply. The mixed model combines the advance reservation and the on-demand requests as a two-stage leasing scheme, which enjoys the complementary strengths of the two stages. Furthermore, taking the competition among MVNOs into account, the amount of inventories can be determined by the non-cooperative inventory game (see [14] and references therein). In our work, we assume that the spectrum inventory decisions for MVNOs have been made by certain means and only focus on the pricing decisions in Stage II. During Stage II, when making pricing decision, one MVNO considers not only the competition from MNOs, but also the competition from other MVNOs. For the former competition, the pricing strategy can be designed by schemes such as the Cournot game [4], [5], [15], Bertrand game [16], [17], Stackelberg game [7], [18], sequential game [6], Bargaining game [19], and the user-centric market survey [20]. For the latter competition,

the pricing strategy can be investigated such as by the Stackelberg game [21], Bertrand game theory [22], and evolutionary game theory [23]. However, these two competitions occur concurrently in reality, but none of the above works consider two kinds of competitions together. Furthermore, the cooperation among MVNOs can enable MVNOs to obtain more profits than that by the non-cooperation case. Besides, a more realistic situation where the private information (i.e., spectrum inventory) is unknown to other MVNOs should be taken into account. Nevertheless, to our best knowledge, no current works have studied this situation.

During Stage III, EUs select a MNO or MVNO as their service provider according to the price (fixed in Stage II) and service (supported by spectrum inventories which determined in Stage I) that MNOs and MVNOs provide. Users' choice behaviors can be modeled by the evolutionary game theory [24], and the potential subscribers for each MVNO can be inferred through the model.

In this work, we study the pricing strategy for MVNOs to maximize their profits in wireless virtualization communication service markets including MNOs, MVNOs, and EUs. Specifically, the main contributions of this paper are as follows:<sup>1</sup>

- We enfold the price and spectrum inventory factors into the utility function of EUs and the revenue function of MVNOs, respectively, and formulate the decision making problem as a non-cooperative game model for the complete spectrum inventory information scenario.
- We reveal the impacts of the price factor on the revenue of MVNOs with given their inventories, and analyze the Cournot Nash equilibrium (C-NE). We obtain several useful conclusions that provide guidance on pricing decisions for MVNOs. Moreover, we develop an optimal price setting algorithm based on an ordinal potential game to achieve the C-NE prices for all MVNOs.
- We analyze the potential cooperation among the MVNOs and propose three kinds of schemes for payoff allocation. For the proportional segmentation, we prove analytically that MVNOs have strong incentives to cooperate and set price coordinately their prices.
- We propose a Bayesian coalition formation game to model the uncertainty in other MVNOs' spectrum inventory and develop an optimal price setting algorithm based on the Minimum Mean-Square Error to resolve conflicts resulting from the uncertainty.
- We define a Belief Pareto Order (BPO) to characterize the preferences of MVNOs regarding coalition structures based on the expected optimal payoff and develop a BPO-based distributed coalition formation algorithm (BPO-DCF) to achieve a Bayesian-Nash stable solution for pricing competition and cooperation among MVNOs.

The remainder of this paper is organized as follows. Related works is described in Section II. We introduce the system model, including the assumptions and basic notations in Section III. Pricing competition among individual MVNOs is analyzed in

Section IV and we explore the strong incentives to cooperate among MVNOs in Section V. In Section VI, we propose the Bayesian coalition formation game model and develop the BPO-DCF algorithm. The performance evaluation results for the proposed game is presented in Section VII. Finally, Section VIII concludes this paper.

## II. RELATED WORKS

To embark on pricing strategy research, two important models need to be clarified, which are the revenue model and the cost model for MVNOs. For the revenue, the authors in [26] proposed that the revenue of one MVNO can be calculated through multiplying the average revenue per user by the number of the MVNO's customers based on the real option theory with uncertain cash flows. However, this work does not propose the method for obtaining the number of potential customers. In reality, since EUs are free to choose the associated MVNOs according to their expected satisfaction, it is hard for one MVNO to get the exact or approximate number of potential customers. Besides, the existence of MNOs makes the estimation more complicated. The authors in [24] combined the evolutionary game theory with the neutral operator to analyze the evolution of user population associated with each operator, and then derived the market share of each operator in the stationary conditions. For the cost model, the authors in [27] divided the cost of one MVNO into the variable cost and the fixed cost, and the former mainly comes from the access charge by the MNOs. Furthermore, the access charge depends on the substitution or complementary relationship between MNOs and MVNOs. Specifically, in the substitution-based relationship, one MVNO is charged higher than its host MNO's marginal cost. Finally, with the revenue model and cost model, the profit for one MVNO is defined as the difference between the revenue and the cost. These two models lay the foundations for our research.

Some works have studied optimal pricing decisions with incomplete information using the Bayesian game. The authors in [28] considered revenue maximization of selling a digital product in a social network. The private valuation of each agent is sampled from a uniform distribution and agents know only a common distribution about others' private valuations. Then agents make decisions simultaneously based on the common prior. The authors in [29] proposed an economic framework for a duopoly femtocell market, where pricing and spectrum allocation strategies are jointly considered to maximize each wireless service provider's utility. However, the system models in these two works are originally not designed for wireless virtualization communication service markets, which makes the corresponding analysis and results different from our work.

As an effective solution to model the unknown information about players in the cooperation game, the Bayesian coalition formation game (BCFG) was respectively utilized by the authors in [30] based on an information set and by the authors in [31] based on types. In recent years, the BCFG has been applied to wireless communications, i.e., packet delivery [32], Internet of

<sup>1</sup>Part of this paper has been submitted to GLOBECOM2018 for review [25], however, the corresponding part in this paper is improved in terms of the algorithm, simulations, and presentation.

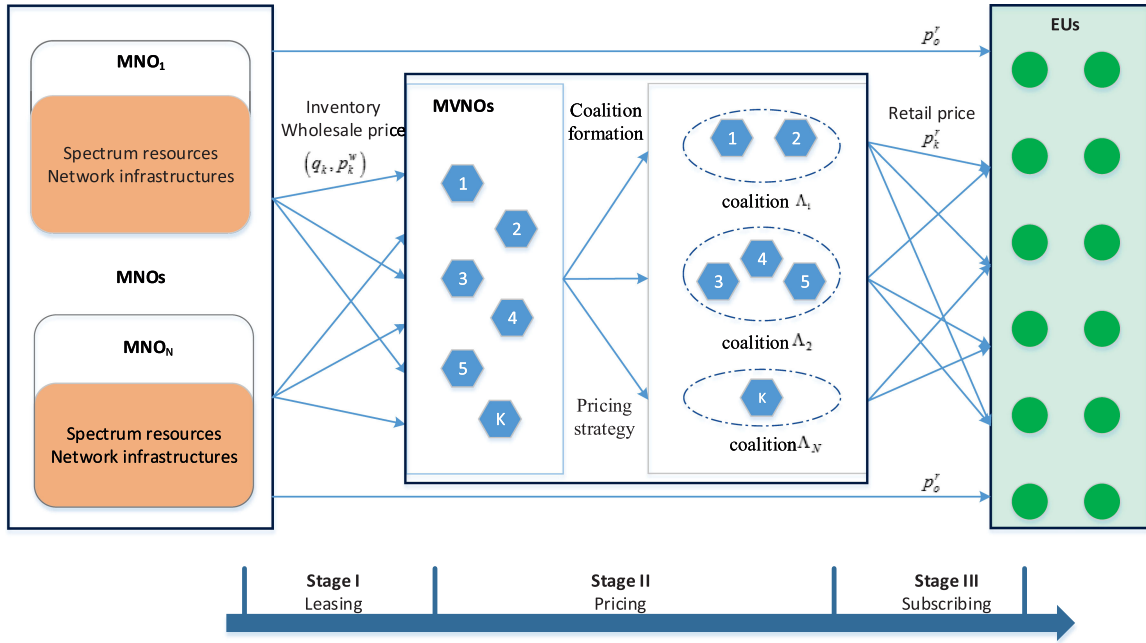


Fig. 1. Three-stage interactions among MNOs, MVNOs and EUs in a wireless virtualization communication service market.

Things [33], resource allocation for device to device communication [34], [35]. In this work, we use the BCFG to facilitate MVNOs to model the unknown inventories of others.

### III. SYSTEM MODEL

#### A. System Overview

We consider a wireless virtualization communication service market (henceforth referred to as *market*), where each of  $K$  separate MVNOs  $\mathcal{V} = \{v_1, \dots, v_K\}$  leases spectrum resources from one of MNOs, and then competes to serve a large number of  $L$  EUs  $\mathcal{U} = \{u_1, \dots, u_L\}$ , as depicted in Fig. 1. The overall service process among MNOs, MVNOs and EUs can be divided into a three-stage dynamic process. In Stage I, MNOs and MVNOs make coordinately spectrum leasing decisions, and then both of them make pricing decisions in Stage II. In Stage III, the EUs are free to choose one of the MVNOs or MNOs based on the perceived satisfaction. To attract more users, some MVNOs may cooperate with each other as a coalition to compete with other non-member MVNOs. In the work, we consider the spectrum inventories of MVNOs as private information and maybe unknown to others. We focus on the pricing competition and assume that each MVN's spectrum inventory is given. However, as shown in the paper, the spectrum inventory has impact on one MVNO's pricing decision. Therefore, both price and spectrum inventory factors should be considered when designing the utility function of MVNOs.

#### B. End User Model

One end user makes a request for data services to that MVNO that offers better service, i.e., higher quality of lower price of service. We assume that more spectrum inventory at one MVNO's disposal generates higher QoS, and thus better users'

satisfaction. However, the EUs' satisfaction will be saturated as the spectrum inventory increases, which is consistent with the economics' principle of diminishing marginal returns. Besides, we assume that the spectrum resource of one MVNO is on average equally shared among its users in the long term [24].

Thus, as [24], we define the utility of each user served by MVNO  $v_k \in \mathcal{V}$  as follows:

$$U_k(L_k, q_k, p_k) = \log \frac{q_k}{L_k} - p_k, \quad (1)$$

where  $q_k$  is the total spectrum inventory at MVNO  $v_k$ 's disposal,  $L_k$  the number of EUs affiliated with MVNO  $v_k$ , and  $p_k$  the retail price of MVNO  $v_k$ , i.e., the charged price for each user affiliating MVNO  $v_k$ . Both the price and utility are measured in monetary unit (MU) during a certain time interval [5], [24].

We assume that each user is rational and free to change from one MVNO to another, if the latter gives a better utility. Besides, each user has a reservation utility of  $U_0$ , which needs to be satisfied when the user consents to pay the MVNO for the service, otherwise the user changes to one MNO. As the selections of users evolve, the market reaches to stationary states where none of users alters its associated MVNO. In the stationary state, MVNOs provide equal utilities to use with each other.

Let  $L_0$  be the number of EUs affiliated with all MNOs. According to the MVNOs' prices and their amounts of spectrum inventory, the stationary states of market share fall into one of the following three possible cases.

- Case I:  $U_i = U_j > U_0$  and  $L_i, L_j > 0, L_0 = 0, \forall v_i, v_j$ ;
- Case II:  $U_i = U_j = U_0$  and  $L_i, L_j > 0, L_0 = 0, \forall v_i, v_j$ ;
- Case III:  $U_k = U_0$  and  $L_k, L_0 > 0, \forall v_k$ .

When Case III occurs, all MVNOs and MNOs provide the same utility to end users, and both MVNOs and MNOs have a certain amount of end users. When Case II occurs, the same as

in Case III, MVNOs and MNOs still provide the same utility for the user, but the difference is that all users have chosen MVNOs instead of MNO for the service. When Case I occurs, MVNOs provide users with higher utility than MNOs. All users choose MVNO as in Case II. Note that two more situations can occur:

- Case IV:  $U_i = U_j = U_0$  and  $L_i, L_j = 0, L_0 > 0, \forall v_i, v_j$ ;
- Case V:  $U_i = U_j < U_0$  and  $L_i, L_j = 0, L_0 > 0, \forall v_i, v_j$ .

When Case IV occurs, the same as Case II and III, MVNOs and MNOs provide the same utility for the user, but the difference is that all users have selected MNOs, and no user chooses MVNOs for services. When Case V occurs, MVNOs provide users with lower utility than MNOs, so all users choose MNOs. In this paper, the service price and spectrum inventory strategies of MVNOs are mainly analyzed. For Case IV and V, MVNOs fail to obtain end users, and thus no gains can be obtained. Therefore, this paper mainly analyzes the strategies of MVNO when Case I, II, and III occur.

Without loss of generality, in the rest of the paper, we assume  $U_0 = 0$ . Thus, the number of users associated with each MVNO is given by the following axiom. The proof is given in Appendix A.

*Axiom 1:* For Case I, the number of users served by MVNO  $v_k$  is given by:

$$\begin{cases} L_k = \frac{Lq_k e^{-p_k}}{\sum_{v_i \in \mathcal{V}} q_i e^{-p_i}} < q_k e^{-p_k}, \\ \sum_{v_i \in \mathcal{V}} L_k = L, \\ \sum_{v_i \in \mathcal{V}} q_i e^{-p_i} > L, \\ L_0 = 0, \end{cases} \quad (2)$$

For Case II, the number of users served by MVNO  $v_k$  is given by:

$$\begin{cases} L_k = q_k e^{-p_k}, \\ \sum_{v_i \in \mathcal{V}} L_k = L, \\ \sum_{v_i \in \mathcal{V}} q_i e^{-p_i} = L, \\ L_0 = 0, \end{cases} \quad (3)$$

For Case III, the number of users served by MVNO  $v_k$  is given by:

$$\begin{cases} L_k = q_k e^{-p_k}, \\ \sum_{v_i \in \mathcal{V}} L_k < L, \\ \sum_{v_i \in \mathcal{V}} q_i e^{-p_i} < L, \\ L_0 = L - \sum_{v_i \in \mathcal{V}} q_i e^{-p_i}. \end{cases} \quad (4)$$

In Case III, EUs are served by MVNOs or MNOs, while EUs are only served by MVNOs in Case I and II. In addition, EUs can only get reservation utilities in Case II and III, while EUs can get a better utility than the reservation value in Case I.

### C. Mobile Virtual Network Operator Model

One MVNO leases  $q_k$  amount of spectrum inventory at wholesale price  $p_k^w$ . In our paper, we consider this access charge as a linear function of spectrum inventory  $q_k$  [36] as follows:

$$\Upsilon_k = \alpha + \beta \times q_k, \quad (5)$$

where  $\alpha$  and  $\beta$  are constant values. In addition, the total cost  $C_{v_k}$  of MVNO  $v_k$  is also considered as the sum of the access charge  $\Upsilon_k$  and the cost unrelated to the spectrum inventory, introduced in [26]  $b_k$  including capital and operating costs, as follow:

$$C_{v_k}(q_k) = \Upsilon_k + b_k. \quad (6)$$

On the other hand, the revenue of MVNO  $v_k$  is the function of its market share  $L_k$  and retail price  $p_k$ :

$$R_k = L_k p_k. \quad (7)$$

Substituting (2), (3) and (4) into (7), the revenue of MVNO  $v_k$  can be expressed as a function of all MVNOs' prices and inventories, given by

$$R_k = \begin{cases} \frac{q_k p_k L e^{-p_k}}{\sum_{v_i \in \mathcal{V}} q_i e^{-p_i}}, & \text{Case I,} \\ q_k p_k e^{-p_k}, & \text{Case II or III.} \end{cases} \quad (8)$$

Finally, we define the profit (or utility) of MVNO  $v_k$  as the difference between its revenue and cost:

$$\pi_k = R_k - C_{v_k}(q_k). \quad (9)$$

## IV. PRICING STRATEGY ANALYSIS

As a profit-making institution, one MVNO should rationally set its retail price to maximize its profit. In this section, we assume that every MVNO's inventory has been given, and each MVNO knows the others' inventory information in the non-cooperation situation.

We regard it as a non-cooperative pricing competition among the MVNOs  $\mathbf{G}_p = (\mathcal{V}, \mathbf{p}, \{\pi_k\})$  as follows:

- **Players:** the set of the  $K$  MVNOs  $\mathcal{V} = \{v_1, \dots, v_K\}$ .
- **Strategies:** every MVNO  $v_k \in \mathcal{V}$  selects its price  $p_k \in [0, p_{\max}]$ ,  $p_{\max} \in \mathbb{R}^+$ .
- **Payoff:** the utility function  $\pi_k : (p_k, \mathbf{p}_{-k}) \rightarrow \mathbb{R}$ , for each MVNO  $v_k$ ,

where  $\mathbf{p}_{-k} = (p_1, \dots, p_{k-1}, p_{k+1}, \dots, p_K)$  is the vector of the prices set by the MVNOs excluding  $v_k$ .

Since each MVNO  $v_k$  selects the price that can maximize its payoff, the best strategy can be given as

$$\max_{p_k} \pi_k(p_k, \mathbf{p}_{-k}). \quad (10)$$

According to the formulation above, there exists the best price choice for each MVNO  $v_k$  with any strategy profiles of other MVNOs. However, the particular characteristic of this game is that each MVNO has two different payoff functions depending on the price profile. Despite this characteristic, the payoff function is continuous and quasi-concave. By solving (10), we can obtain the best reaction function of MVNO  $v_k$ , given by the following Lemma 1. The proof is given in Appendix B. Without affecting the correctness of the upcoming conclusions, for the simplicity of the expressions, we treat the revenue  $R_k$  maximization instead of  $\pi_k$  in the rest of this section.

This best response price constitutes a reaction curve to the prices set by the other MVNOs. The Cournot Nash Equilibrium (C-NE) of the proposed game is a pricing strategy profile where none of the MVNOs can improve its payoff by changing its price and the specific definition is given in Definition 1.

*Lemma 1:* The best response price of the MVNO  $v_k$  is:

$$p_k^* = \begin{cases} \mu_k, & \text{if } (\mathbf{p}_k, \mathbf{q}_k) \text{ s.t. Case I,} \\ \eta_k, & \text{if } (\mathbf{p}_k, \mathbf{q}_k) \text{ s.t. Case II,} \\ 1, & \text{if } (\mathbf{p}_k, \mathbf{q}_k) \text{ s.t. Case III,} \end{cases} \quad (11)$$

where  $\mu_k$  is the solution to  $(\mu_k - 1)e^{\mu_k} = \frac{q_k}{\sum_{v_i \in \mathcal{V}/v_k} q_i e^{-p_i}}$ , and  $\eta_k = \log \frac{q_k}{L - \sum_{v_i \in \mathcal{V}/v_k} q_i e^{-p_i}}$ .

*Definition 1:* A Cournot Nash Equilibrium for the non-cooperative pricing game  $\mathbf{G}_p = (\mathcal{V}, \mathbf{p}, \{\pi_k\})$  is a  $K$ -tuple vector  $(p_1^*, p_2^*, \dots, p_K^*)$  such that,

$$R_k(p_k^*, \mathbf{p}_{-k}^*) = \max_{p_k} R_k(p_k, \mathbf{p}_{-k}^*), \forall v_k \in \mathcal{V}. \quad (12)$$

In fact, the pricing competition game is a finite ordinal potential game [37], and therefore, it has the pure C-NE. Analytically, we can obtain the optimal price vector  $\mathbf{p}$  by solving the best response equations of all the MVNOs jointly. Practically, the game is played repeatedly and any finite improvement path can guarantee the convergence to the C-NE. Next, we will analyze the best response for MVNO  $v_k$  according the current others' prices and inventories.

On the condition that  $\sum_{v_i \in \mathcal{V}/v_k} q_i e^{-p_i} \geq L$ , denoted as **C1**, only Case I can occur, then  $p_k^* \in (0, P_{\max}]$ . Moreover, since  $\frac{\partial R_k}{\partial p_k} \geq 0$  at the points that satisfy  $p_k \leq 1$ ,  $p_k^* \in [1, P_{\max}]$ . If  $(P_{\max} - 1)e^{P_{\max}} \leq \frac{q_k}{\sum_{v_i \in \mathcal{V}/v_k} q_i e^{-p_i}}$ ,  $\frac{\partial R_k}{\partial p_k} \geq 0$  at  $(0, P_{\max}]$ , thus  $p_k^* = P_{\max}$ . Otherwise,  $p_k^*$  is the solution to  $(p_k - 1)e^{p_k} = \frac{q_k}{\sum_{v_i \in \mathcal{V}/v_k} q_i e^{-p_i}}$ .

In contrast, for the situation where  $\sum_{v_i \in \mathcal{V}/v_k} q_i e^{-p_i} < L$ , if  $q_k e^{-P_{\max}} + \sum_{v_i \in \mathcal{V}/v_k} q_i e^{-p_i} > L$ , denoted as **C2**, similar to **C1**, only Case I may occur and the best response price is the same with that in **C1**. Otherwise, if  $q_k e^{-P_{\max}} + \sum_{v_i \in \mathcal{V}/v_k} q_i e^{-p_i} = L$ , denoted as **C3**, Case I can occur at  $(0, P_{\max})$  and Case I can occur at  $P_{\max}$ . Since  $\sum_{v_i \in \mathcal{V}} q_i e^{-p_i} > L$  for Case I,  $\frac{q_k p_k L e^{-p_k}}{\sum_{v_i \in \mathcal{V}} q_i e^{-p_i}} < q_k p_k L e^{-p_k}$ ,  $\forall p_k \in (0, P_{\max}]$ , the best response will occur at  $P_{\max}$  in Case II.

Furthermore, for the situation where  $q_k e^{-P_{\max}} + \sum_{v_i \in \mathcal{V}/v_k} q_i e^{-p_i} > L$ , if  $q_k + \sum_{v_i \in \mathcal{V}/v_k} q_i e^{-p_i} > L$ , denote as **C4**, Case I can occur at  $(0, \eta_k)$ , Case II can occur at  $\eta_k$ , and Case III can occur at  $(\eta_k, P_{\max}]$ , where  $\eta_k = \log \frac{q_k}{L - \sum_{v_i \in \mathcal{V}/v_k} q_i e^{-p_i}}$ . Similar to **C3**, the revenue resulting from the best response in Case I is less than ones in Case II and Case III, the best response will occur in Case II and Case III. If  $\eta_k \geq 1$ ,  $\frac{\partial R_k}{\partial p_k} \geq 0$  at  $p_k \in [\eta_k, P_{\max}]$ . Therefore,  $R_k$  is monotonically decreasing function of  $p_k$ , and thus the best response is  $\eta_k$ . Otherwise, the best response will occur at the point where  $\frac{\partial R_k}{\partial p_k} = 0$ , and therefore  $p_k^* = 1$ . If  $q_k + \sum_{v_i \in \mathcal{V}/v_k} q_i e^{-p_i} \leq L$ , denoted as **C5**, only Case III can occur at  $(0, P_{\max}]$ . Thus, the best response will occur at the point where  $\frac{\partial R_k}{\partial p_k} = 0$ , and therefore  $p_k^* = 1$ .

Each MVNO adjust its price to the best response to the current others' prices. The above update process will continue until no MVNO will change its price. Consequently, we obtain

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**Algorithm 1:** Optimal Price Setting Based on Ordinal Potential Game.

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**Input:** A set of  $K$  MVNOs  $\mathcal{V}$ , each MVNO's inventory  $q_k$ , the number of EUs  $L$ , and the highest price  $P_{\max}$ .

- 1: **Initial State.** The initial price vector of all the MVNOs is  $\mathbf{p} = (p_1, \dots, p_K)$ .
- 2: **loop**
- 3: Any MVNO updates its price  $\mathbf{p}_k$  as follows:
- 4: **if C1 or C2 holds then**
- 5: **if**  $(P_{\max} - 1)e^{P_{\max}} \leq \frac{q_k}{\sum_{v_i \in \mathcal{V}/v_k} q_i e^{-p_i}}$  **holds then**
- 6:     Update the price to the  $P_{\max}$ ,
- 7: **else**
- 8:     Update the price to the one which is the solution to  $(p_k - 1)e^{p_k} = \frac{q_k}{\sum_{v_i \in \mathcal{V}/v_k} q_i e^{-p_i}}$ .
- 9: **end if**
- 10: **else if C3 holds then**
- 11:     Update the price to the  $P_{\max}$ .
- 12: **else if C4 holds and**  $\eta_k = \log \frac{q_k}{L - \sum_{v_i \in \mathcal{V}/v_k} q_i e^{-p_i}} \geq 1$  **then**
- 13:     Update the price to the  $\eta_k$ ,
- 14: **else**
- 15:     Update the price to 1.
- 16: **end if**
- 17: **end loop** while the price vector  $\mathbf{p}$  no longer changes.

**Output:** the optimal price vector  $\mathbf{p}$ .

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the optimal price for each MVNO through the non-cooperative game, and the overall solution is demonstrated in Algorithm 1.

It can be observed through the above analysis that the C-NE will exist in three main situations including Case I, Case II, and Case III. For Case I, the C-NE exists at the point that all the partials are zeros or at least one MVNO set the highest price  $P_{\max}$ . For Case II, the C-NE exists at the point that  $\eta_k, \forall v_k \in \mathcal{V}$  or at least one MVNO set the highest price  $P_{\max}$ . For Case III, obviously only one C-NE will occur, i.e., all the MVNOs set the price to 1.

Actually, the occurrence of the situations depends on the spectrum inventories  $\mathbf{q}$  of all MVNOs and the number of EUs  $L$ . When MVNOs lease a small amount of spectrum resources, MVNOs only can offer marginal services to a small amount of users, and the rest of users will select MNOs to provide service, and therefore the C-NE occurs in Case III. As the inventories increase, MVNOs can induce users to deviate from MNOs and provide more users with services until MVNOs can serve all users. However, MVNOs only offer marginal services and all the users only obtain the reservation utilities, and therefore the C-NE occurs in Case II. With the further increase of inventories, MVNOs can offer better services than reservation utilities to users and compete with other MVNOs to attract more users. Therefore, the C-NE occurs in Case III.

In particular, for Case I the optimal price for each MVNO can be obtain by solving an equation which is given by Theorem 1 and the proof is shown in Appendix C.

*Theorem 1:* The C-NE price for each MVNO  $v_k$  is reached if and only if the following equation holds,

$$\begin{cases} \sum_{v_i \in \mathcal{V}} q_i e^{-p_i^*} > L, & (p_k^*, \mathbf{P}_{-k}^*) \text{ s.t. Case I,} \\ \sum_{v_i \in \mathcal{V}} q_i e^{-p_i^*} = L, & (p_k^*, \mathbf{P}_{-k}^*) \text{ s.t. Case II,} \\ p_k^* = 1, \text{ and } \sum_{v_i \in \mathcal{V}} q_i < eL, & (p_k^*, \mathbf{P}_{-k}^*) \text{ s.t. Case III,} \end{cases} \quad (13)$$

and the corresponding optimal revenue at the C-NE price is

$$R_k^* = \begin{cases} L(p_k^* - 1), & \text{Case I} \\ q_k p_k^* e^{-p_k^*}, & \text{Case II} \\ q_k \frac{1}{e}, & \text{Case III} \end{cases} \quad (14)$$

where for first case, the following equation array holds,

$$\begin{cases} (p_1^* - 1)e^{p_1^*} = \frac{q_1}{\sum_{v_i \in \mathcal{V}/v_1} q_i e^{-p_i^*}}, & \dots, \\ (p_K^* - 1)e^{p_K^*} = \frac{q_K}{\sum_{v_i \in \mathcal{V}/v_K} q_i e^{-p_i^*}}. \end{cases} \quad (15)$$

As mentioned before, the spectrum inventories of MVNOs affect pricing decisions. If one MVNO has more inventory, the MVNO can increase its price to make more profits. We give this observation in Proposition 1 and the proof is shown in Appendix D.

*Proposition 1:* When one MVNO increases its inventory with the others constant, the MVNO's C-NE price increases while the others' decrease.

In addition, if two MVNOs have the same inventories, these two MVNOs will set the same price. This observation is shown in Proposition 2 and the proof is given in Appendix E.

*Proposition 2:* If two MVNOs have the same inventory  $q_i = q_j$ , they have the same C-NE price  $p_i^* = p_j^*$ .

In particular, if all the MVNOs have the same inventory, the MVNOs will have the same price decision and the price can be obtained by the following corollary.

*Corollary 1:* When all the MVNOs have the same inventory  $q_k = q, \forall v_k \in \mathcal{V}$ , the MVNOs have the same C-NE price  $p_k^* = p$  and the price is

$$p = \begin{cases} \frac{K}{K-1}, & q > \frac{L}{K} e^{\frac{K}{K-1}}, \\ \eta, & q e^{-\eta} = \frac{L}{K}, \\ 1, & q < \frac{L}{K} e, \end{cases} \quad (16)$$

and they have the same revenue  $R_k^* = R^*, \forall v_k \in \mathcal{V}$ , given by

$$R^* = \begin{cases} \frac{L}{K-1}, & q > \frac{L}{K} e^{\frac{K}{K-1}} \\ q \eta e^{-\eta}, & q e^{-\eta} = \frac{L}{K}, \\ q \frac{1}{e}, & q < \frac{L}{K} e. \end{cases} \quad (17)$$

## V. COOPERATION AMONG MVNOs

To make more profits, MVNOs will form coalitions to compete with others [38]. The key issue is how to distribute benefits among member MVNOs [39]. In this section, we study three kinds of cooperation strategies, i.e., proportional segmentation (PS), logarithmic segmentation (LS) and equivalent segmentation (ES). For PS, MVNOs allocate payoffs according to

the amount of spectrum inventories. The payoff ratio among MVNOs equals the spectrum inventory ratio. For LS, the total revenue of one coalition of MVNOs is distributed according to the logarithm of spectrum inventories. The payoff ratio among MVNOs equals the ratio of the inventory plus a constant. For ES, each MVNO obtain the same revenue with other member MVNOs. In what follows, we derive the specific price and revenue of each MVNO with respect to these three cooperation strategies. Note that we only present the derivation for Cases I and II defined in Section III, since for Case III, all the MVNOs set its price equal to 1 as given in Lemma 1 and its revenue is  $q_k \frac{1}{e}$  as given in Theorem 1.

### A. Proportional Segmentation

For the cooperation strategy of proportional segmentation, as we have observed that all the member MVNOs will set the same price. However, unlike the situation in non-cooperative game, the MVNOs have different revenues. Let  $p_k = p$ , for Case II, since  $\sum_{v_i \in \mathcal{V}} q_i e^{-p_i} = L$ , we can get  $p = \log \frac{\sum_{v_i \in \mathcal{V}} q_i}{L}$ . Thus, we have

$$R_k = q_k p_k e^{-p_k} = p e^{-p} q_k = \frac{L q_k}{\sum_{v_i \in \mathcal{V}} q_i} \log \frac{\sum_{v_i \in \mathcal{V}} q_i}{L}. \quad (18)$$

Let  $R_{II}$  denote the sum of all MVNOs' revenues, and we can get that

$$R_{II} = \sum_{v_i \in \mathcal{V}} R_k = \sum_{v_i \in \mathcal{V}} p e^{-p} q_k = L p = L \log \frac{\sum_{v_i \in \mathcal{V}} q_i}{L}. \quad (19)$$

In contrast, for Case I, since  $\sum_{v_i \in \mathcal{V}} q_i e^{-p_i} > L$ , we have  $p < \log \frac{\sum_{v_i \in \mathcal{V}} q_i}{L}$ . Besides, the price of one MVNO is not greater than the allowable maximization  $P_{\max}$ , and thus  $p = \min\{\log \frac{\sum_{v_i \in \mathcal{V}} q_i}{L} - \delta, P_{\max}\}$ . Furthermore, the revenue can be obtained by the following equation

$$\begin{aligned} R_k &= \frac{q_k p_k L e^{-p_k}}{\sum_{v_i \in \mathcal{V}} q_i e^{-p_i}} = \frac{L q_k}{\sum_{v_i \in \mathcal{V}} q_i} p, \\ &= \frac{L q_k}{\sum_{v_i \in \mathcal{V}} q_i} \min \left\{ \log \frac{\sum_{v_i \in \mathcal{V}} q_i}{L} - \delta, P_{\max} \right\}. \end{aligned} \quad (20)$$

Let  $R_I$  denote the sum of all MVNOs' revenues and we can get that

$$\begin{aligned} R_I &= \sum_{v_i \in \mathcal{V}} R_k = \sum_{v_i \in \mathcal{V}} \frac{L q_k}{\sum_{v_i \in \mathcal{V}} q_i} p, \\ &= L p = L \min \left\{ \log \frac{\sum_{v_i \in \mathcal{V}} q_i}{L} - \delta, P_{\max} \right\}. \end{aligned} \quad (21)$$

### B. Logarithmic Segmentation

For the cooperation strategy of logarithmic segmentation, as we have observed that all the member MVNOs have the same served users. For Case II, since  $L_k = q_k e^{-p_k}$  and  $\sum_{v_i \in \mathcal{V}} L_k = L$ , we have  $q_k e^{-p_k} = \frac{L}{K}, \forall v_k \in \mathcal{V}$ . Then, we can get the price  $p_k = \log \frac{K q_k}{L}$ . Moreover, the revenue for MVNO  $v_k$  is given by

TABLE I  
THE COOPERATION STRATEGY PARAMETERS FOR CASE II

Cooperation Strategy	price ( $p_k$ )	revenue ( $R_k$ )	sum revenue ( $(\sum R_k)$ )
PS	$\log \frac{\sum_{v_i \in \mathcal{V}} q_i}{L}$	$\frac{Lq_k}{\sum_{v_i \in \mathcal{V}} q_i} \log \frac{\sum_{v_i \in \mathcal{V}} q_i}{L}$	$L \log \frac{\sum_{v_i \in \mathcal{V}} q_i}{L}$
LS	$\log \frac{Kq_k}{L}$	$\frac{L}{K} \log \frac{Kq_k}{L}$	$\frac{L}{K} \sum_{v_i \in \mathcal{V}} \log \frac{Kq_k}{L}$
ES	$-W_0\left(-\frac{\sigma}{q_k}\right)$ $\sum_{v_k \in \mathcal{V}} q_k e^{W_{-1}\left(-\frac{\sigma}{q_k}\right)} = L$	$\sigma$	$K\sigma$

$R_k = \frac{L}{K} p_k = \frac{L}{K} \log \frac{Kq_k}{L}$ . Thus, we can the sum revenue  $R_{II} = \frac{L}{K} \sum_{v_i \in \mathcal{V}} \log \frac{Kq_k}{L}$ .

In contrast, for Case I, MVNO  $v_k$  serves the same number of users with Case II. However, according to the definition of Case I in Section III, unlike Case II  $\log \frac{q_k}{L_k} - p_k > 0$ . Let  $\log \frac{q_k}{L_k} - p_k = a$ , where  $a$  is a positive constant number. Then, we can get the price  $p_k = \log \frac{Kq_k}{L} - a$  for MVNO  $v_k$ . In addition, as mentioned before, the price of one MVNO is not greater than the allowable maximization  $P_{\max}$ , and thus the price is given by  $p_k = \min\{\log \frac{Kq_k}{L} - \delta, P_{\max}\}$ . Consequently, the revenue for MVNO  $v_k$  and the overall revenue for all the member MVNOs are given by  $R_k = \frac{L}{K} p_k = \min\{\frac{L}{K} \log \frac{Kq_k}{L} - \delta, \frac{L}{K} P_{\max}\}$  and  $R_I = \min\{\frac{L}{K} \sum_{v_i \in \mathcal{V}} \log \frac{Kq_k}{L} - \delta, LP_{\max}\}$ , respectively.

### C. Equivalent Segmentation

Though the payoff is distributed equally among MVNOs, the payoff can not be obtained directly since the total revenue is unknown. We assume the revenue for each MVNO is the same  $\sigma$ . Since  $R_k = q_k p_k e^{-p_k} = \sigma, \forall v_k \in \mathcal{V}$  according to Theorem 1, we have  $-p_k e^{-p_k} = -\frac{\sigma}{q_k}$ . Therefore, we can get the price for MVNO  $v_k$   $p_k = -W_0\left(-\frac{\sigma}{q_k}\right)$ , where  $W_0(\cdot)$  is the lower principle branch of the real-valued Lambert-W function [40]. In addition, since  $\sum_{v_k \in \mathcal{V}} q_k e^{-p_k} = L$  according to Theorem 1, we have  $\sum_{v_k \in \mathcal{V}} q_k e^{W_{-1}\left(-\frac{\sigma}{q_k}\right)} = L$ . By solving the equation, we can obtain  $\sigma$ . Thus, the sum revenue of all the member MVNOs is  $K\sigma$ .

In summary, the price and revenue for MVNO  $v_k$  and the sum revenue for all the member MVNOs as to Case II are shown in Table I. The results for Case I are omitted due to space limitations. Specifically, for the situation where all MVNOs have the same spectrum inventory  $q_k = q$ , the proportional segmentation is equivalent to the logarithmic segmentation.

In what follows, we study the impacts of coalition structure on the pricing strategies for MVNOs. For ease of description, we only discuss the cooperation strategy of the proportional segmentation where all the member MVNOs set the same price and only discuss Case I. Thus, we call coalition price the one set by all the member coalition cooperatively. Specifically, a group of MVNOs form a coalition, denoted with  $\Lambda_n, \Lambda_n \subseteq \mathcal{V}$ , and set the same price  $p_{\Lambda_n}$ . When there are more than one coalitions, each coalition compete with other coalitions for selecting the retail prices to yield the higher payoffs. Clearly, the mechanics of this competition are the same as when different individual MVNOs compete with each other. Let  $\mathbf{q}_{\Lambda_n} = \sum_{v_k \in \Lambda_n} q_k$  the

total spectrum inventory of the participating MVNOs of coalition  $\Lambda_n$ . In other words, a coalition acts as a single virtual MVNO with  $\mathbf{q}_{\Lambda_n}$  amount of spectrum inventory. According to (8), the revenue of MVNO  $v_k$  that participates in coalition  $\Lambda_n$  is

$$R_k(p_{\Lambda_n}) = \begin{cases} \frac{q_k p_{\Lambda_n} L}{\mathbf{q}_{\Lambda_n} + e^{p_{\Lambda_n}} \sum_{v_i \in \mathcal{V} / \Lambda_n} q_i e^{-p_i}}, & p_{\Lambda_n} < \eta_k(\Lambda_n), \\ q_k p_{\Lambda_n} e^{-p_{\Lambda_n}}, & p_{\Lambda_n} \geq \eta_k(\Lambda_n), \end{cases} \quad (22)$$

where  $\eta_k(\Lambda_n) = \log \frac{\mathbf{q}_{\Lambda_n}}{L - \sum_{v_i \in \mathcal{V} / \Lambda_n} q_i e^{-p_i}}$  that depends on the specific coalition  $\Lambda_n$ .

As a profit-making entity, each member MVNO will propose the coalition price to maximize its payoff. However, the most uncontroversial method is to set the coalition price to maximize the overall revenue of all the member MVNOs. In fact, the price set from a personal perspective is the same with the one set from the coalition perspective. We give this conclusion in Theorem 2 and the proof is given in Appendix F.

*Theorem 2:* The coalition price proposed by one MVNO to maximize its payoff is consistent with the one set by maximizing the coalition's aggregate revenue.

The formation of two or more MVNOs into one new coalition will have an effect on the prices of these MVNOs and also have impacts on the prices of others. In addition the overall revenue of one coalition a close relationship with the coalition's price. We give these conclusions in Theorem 3 and the proof is given in Appendix G.

*Theorem 3:* When two or more MVNOs form to a coalition, the equilibrium prices of all MVNOs increase. There is a direct proportionality between aggregate revenue of MVNOs participating in a coalition and their prices.

However, one MVNO will have lower or higher revenue when it participates in different coalition. The resulted revenue is related with the overall spectrum inventory of the participating coalition, as shown in Theorem 4.

*Theorem 4:* In the Cournot NE, MVNOs in coalition  $(\Lambda_j : \Lambda)$  get lower revenue per unit spectrum compared to the MVNOs in coalition  $(\Lambda_j : \Lambda)$ , if  $\mathbf{q}_{\Lambda_j} > \mathbf{q}_{\Lambda}$ .

Theorem 4 along with Theorem 3 indicates that some MVNOs may not participate in a coalition and act independently. This is due to the fact that MVNOs with small amount of spectrum inventory may obtain lower revenue when participating with a coalition with large  $\mathbf{q}_{\Lambda_n}$ . In this situation, although the aggregate revenue of the coalition increase (after joining), the individual share of the newly joining MVNO may decrease.

Up to now, we have the pricing decision and cooperation strategy under the assumption that the spectrum inventory of

each MVNO is known to others. However, in fact these information are private and will not share with each other even though they cooperate as a coalition. In the next section, we will propose Bayesian coalition formation game to consider this pricing competition problem with incomplete and uncertain information.

## VI. BAYESIAN COALITION FORMATION GAME AMONG MVNOS

In this section, we consider the pricing decision and cooperation strategy when the spectrum inventory of each MVNO is unknown to others. We use Bayesian coalition formation game (BCFG) to model this uncertainties [31], propose belief Pareto order to model the preference of each MVNO to coalitions and then develop an algorithm to solve the formulated problem.

### A. The Model

*Definition 2:* The Bayesian coalition formation game  $\mathcal{G} = \langle \mathcal{V}, \Theta, (B^k), (\mathcal{A}^{\Lambda_n}), \varpi \rangle$  is a coalition formation game, characterized by

- a set of **players**  $\mathcal{V}$ .
- a finite set of possible **types**  $\Theta^k$  for each player  $v_k \in \mathcal{V}$ . Let  $\Theta = \times_{v_k \in \mathcal{V}} \Theta^k$  denote the set of all players' type profiles. For any coalition  $\Lambda_n \subseteq \mathcal{N}$ ,  $\Theta^{\Lambda_n} = \times_{v_k \in \Lambda_n} \Theta^k$ , and for any  $v_k \in \mathcal{V}$ ,  $\Theta^{-k} = \times_{j \neq k} \Theta^j$ . Each player  $v_k$  knows its own type  $\theta^k$ , but not those of other players  $\theta^{-k} \in \Theta^{-k}$ .
- a player  $v_k$ 's **beliefs**  $B^k$ , which comprise a joint distribution over  $\Theta^{-k}$ , where  $B^k(\theta^{-k})$  is the probability that  $v_k$  assigns to other players having type profile  $\theta^{-k}$ . A function  $B^k(\theta^{\Lambda_n})$  indicates a marginal value of  $B^k$  over any coalition  $\Lambda$  with members' types  $\theta^{\Lambda_n} = \{\theta^k | v_k \in \Lambda_n \in \Theta^{\Lambda_n}\}$ . For ease of notation, we let  $B^k(\theta^k)$  refer to  $v_k$ 's "belief" about its own type (assigning probability 1 to its actual type and 0 to all others).
- a finite set of coalitional **actions**  $\mathcal{A}^{\Lambda_n}$  that a coalition  $\Lambda_n$  has available to it.
- a **payoff allocation** function  $\varpi$ , which assigns to each player a payoff  $\phi_k(\Lambda_n; \Lambda)$ .

We can model the pricing competition among MVNOS as a BCFG and the components of it for our problem are described below. The players are the MVNOS. The MVNO's type  $\theta^k = q_k$  indicates its private decision about the traffic inventory. Each MVNO cannot know the types of other MVNOS. The coalitional action of a coalition  $\Lambda_n$  corresponds to the retail price set by all MVNOS in coalition  $\Lambda_n$ .  $\phi_k(\Lambda_n; \Lambda, B^k) = R_k^*$  is the expected revenue of any MVNO  $v_k$  in coalition  $\Lambda_n$  within a partition  $\Lambda$  given the beliefs of MVNO  $v_k$  about the types of all players in the coalition  $\Lambda_n$  and the coalitional action  $p_{\Lambda_n}$ . Here we stress that, according to the nature of market, the coalitional actions are *completely observable*: all members of coalition  $\Lambda_n$  can not only observe the actions of this coalition but also know the actions of other coalitions.

The non-cooperative game among the  $\{\Lambda_1, \dots, \Lambda_N\}$ ,  $N > 1$  coalitions is identical to the pricing competition game with  $N$  MVNOS where  $q_{\Lambda_n}, \forall n = 1, \dots, N$ . According to (9) and (22), with given types of all MVNOS  $\theta$ , namely traffic inventory  $\mathbf{q} =$

$(q_1, \dots, q_K)$ , the revenue of MVNO  $v_k$  participating coalition  $\Lambda_n$  is:

$$R_k(\Lambda_n; \Lambda, \mathbf{p}_\Lambda | \mathbf{q}) = \frac{q_k p_{\Lambda_n} L e^{-p_{\Lambda_n}}}{\sum_{i=1}^N \left( \sum_{v_j \in \Lambda_i} q_j \right) e^{-p_{\Lambda_i}}}. \quad (23)$$

Since each MVNO cannot know the exact types of all others, it can only estimate (based on its belief  $B^k$ ) the *expected payoff* using

$$\phi_k(\Lambda_n; \Lambda, \mathbf{p}_\Lambda, B^k) = \sum_{q_{-k} \in \mathbb{Q}_{-k}} B^k(q_{-k}) R_k(\Lambda_n; \Lambda, \mathbf{p}_\Lambda | \mathbf{q}), \quad (24)$$

where  $\mathbb{Q}_{-k}$  is the type space for the MVNOS except  $v_k$ . Given the other coalitions' prices, the *expected optimal payoff* (EOP) for each MVNO can be obtained by maximizing (24) among any possible values  $p_k$  as (25) shows.

$$\bar{\phi}_k = \max_{p_k} \phi_k(\Lambda_n; \Lambda, \mathbf{p}_\Lambda, B^k). \quad (25)$$

However, since the optimal points of expected payoffs for every MVNOS within the same coalition maybe different, they may propose different coalition's price. In this case, each MVNO calculates the *negotiated optimal payoffs* NOPs based on others' proposed prices, denoted as  $\{\phi_{k,1}, \phi_{k,2}, \dots, \phi_{k,k-1}, \phi_{k,k+1}, \dots, \phi_{k,K_n}\}$ , where  $\phi_{k,j}$  is the NOP for MVNO  $v_k$  based on MVNO  $v_j$ 's proposed price. For each MVNO  $v_k$ , there is difference between its EOP  $\bar{\phi}_k$  and NOP  $\phi_{k,j}$  based on  $v_j$ 's proposed price. Thus, we can get the mean-square error (MSE) for all MVNOS within coalition  $\Lambda_n$ , shown in

$$\text{MSE}_{\Lambda_n} = \frac{1}{K_n} \sum_{i=1}^{K_n} (\bar{\phi}_i - \phi_{i,k})^2, \quad (26)$$

where  $K_n$  is the size of coalition  $\Lambda_n$  and  $\phi_{k,k} = \bar{\phi}_k$ .

We propose the coalition price is the one that minimizes the mean-square error, as shown below:

$$p_{\Lambda_n} = \arg \min_{p_{k, \Lambda_n}} \left\{ \frac{1}{K_n} \sum_{i=1}^{K_n} (\bar{\phi}_i - \phi_{i,k})^2 \right\}. \quad (27)$$

When one coalition changes its price, the others update their prices correspondingly. This will lead to further updates by others. The update process terminates until no coalition changes its price, and the overall process is illustrated in Algorithm 2.

According to Algorithm 2 the optimal price  $p_{\Lambda_n}^*$  can be found for each coalition  $\Lambda_n$  and the price vector is  $\mathbf{p}_\Lambda^* = (p_{\Lambda_1}^*, \dots, p_{\Lambda_N}^*)$  when MVNOS in all coalitions set their best response prices  $p_{\Lambda_n}^*$  taking into account the actions of the other coalitions. Each MVNO  $v_k$  can obtain its payoff based on this optimal price vector and we call this *agreed optimal payoff*. Note that the price and revenue of each MVNO at the C-NE, depends not only on the size of its coalition, but also on the coalition structure that are formed by the other MVNOS. Moreover, we assume that there is no central entity coordinating the interactions of MVNOS and each MVNO choose a coalition that the player prefers to be a member of according to its potential optimal payoff.



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**Algorithm 2:** Optimal Price Setting Based on Minimum Mean-Square Error.

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- 1: **Input.** set of MVNOs  $\mathcal{V}$ , partitioned by  $\Lambda = \{\Lambda_1, \dots, \Lambda_I\}$ , each MVNO's type  $q$ , type space  $\mathbb{Q}$ , each MVNO's belief about others' types  $B^k(\mathbf{q}_{-k})$  and the number of EUs  $L$ .
- 2: **Initial State.** The initial price vector is  $\mathbf{p} = (p_{\Lambda_1}, \dots, p_{\Lambda_N})$ .
- 3: **loop**
  - each coalition updates its price  $p_{\Lambda_n}$  as follows:
    - each MVNO  $v_k \in \Lambda_n$  proposes a candidate price and has its *expected optimal payoff*  $\bar{\phi}_k$ .

$$p_{k, \Lambda_n} = \arg \max_{P_{\Lambda_n}} \phi_k(\Lambda_n; \Lambda, \mathbf{p}_\Lambda, B^k).$$

- each MVNO calculates its *negotiated optimal payoff* based on other members' proposed prices  $\{\phi_{k,1}, \phi_{k,2}, \dots, \phi_{k,k-1}, \phi_{k,k+1}, \dots, \phi_{k,K_n}\}$ .
- the price is updated by

$$p_{\Lambda_n} = \arg \min_{P_{k, \Lambda_n}} \left\{ \frac{1}{K_n} \sum_{i=1}^{K_n} (\bar{\phi}_i - \phi_{i,k})^2 \right\}.$$

- 4: **end loop** when the price vector  $\mathbf{p}$  no longer change.
  - 5: **Output** the optimal price vector  $\mathbf{p}$ .
- 

### B. Coalition Formation

To propose a protocol for dynamic coalition formation, we define two concepts *preference* and *collection of coalitions*.

*Definition 3:* A collection of coalitions in the grand coalition  $\mathcal{V}$ , denoted  $\mathcal{S}$ , is defined as the set  $\mathcal{S} = \{S_1, \dots, S_I\}$  of mutually disjoint coalitions  $S_i \subseteq \mathcal{V}$ , and this group of disjoint coalitions  $S_i$  of  $\mathcal{V}$  not necessarily span all MVNOs of  $\mathcal{V}$ .

Rather than the aggregate revenue of the MVNOs, we have to consider individual revenues. Therefore, we define Belief Pareto Order (BPO)  $\triangleright_{\text{BPO}}$  as a comparison metric between two collections of coalitions.

*Definition 4:* Given two collections  $\mathcal{R}$  and  $\mathcal{S}$  of the same players,  $\mathcal{R}$  is preferred over  $\mathcal{S}$  by Belief Pareto Order, denoted as  $\mathcal{R} \triangleright_{\text{BPO}} \mathcal{S}$ , if at least one player in  $\mathcal{R}$  (i.e.,  $\mathcal{M} \subseteq \mathcal{R}, |\mathcal{M}| \geq 1$ ) believes that its expected optimal payoff is improved (i.e.,  $\phi_k^*(\mathcal{R}, B^k) > \phi_k^*(\mathcal{S}, B^k), \forall v_k \in \mathcal{M}$ ) and all other players in  $\mathcal{R}$  (i.e.,  $\mathcal{R}/\mathcal{M}$ ) believe that they are not worse off (i.e.,  $\phi_i^*(\mathcal{R}, B^i) \geq \phi_i^*(\mathcal{S}, B^i), \forall v_i \in \mathcal{R}/\mathcal{M}$ ).

Using Belief Pareto Order, merging and splitting rules for forming and breaking coalitions can be defined as follows:

*Definition 5. Merge Rule:* Any collection of disjoint coalitions  $\mathcal{S} = \{S_1, \dots, S_I\}$  may be merged into a single coalition  $\mathcal{T} = \bigcup_{i=1}^I S_i$  whenever the new coalition  $\mathcal{T}$  is preferred by all the players over the previous collection of coalitions  $\mathcal{S}$  according to the Belief Pareto Order; i.e. where  $\mathcal{T} \triangleright_{\text{BPO}} \mathcal{S}$ , therefore,  $\mathcal{S} \rightarrow \mathcal{T}$ .

*Definition 6. Split Rule:* A single coalition  $\mathcal{T}$  may be split into a collection of disjoint coalitions  $\mathcal{S} = \{S_1, \dots, S_I\}$  whenever the new split form  $\mathcal{S}$  is preferred by the players over the

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**Algorithm 3:** Distributed Coalition Formation Algorithm Based on the Belief Pareto Order.

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- 1: Given set of MVNOs  $\mathcal{V} = \{v_1, \dots, v_K\}$ , each MVNO's type  $q$ , type space  $\mathbb{Q}$ , each MVNO's belief about others' types  $B^k(\mathbf{q}_{-k})$  and the number of EUs  $L$ .
  - 2: **Initial State.** The MVNOs is partitioned by  $\Lambda = \{\Lambda_1, \dots, \Lambda_I\}$  (At the beginning  $\Lambda = \mathcal{V}$ ).
  - 3: **loop**
    - Obtain the optimal price vector  $\mathbf{p}^*$  according Algorithm 2 with input  $\Lambda$ .
    - $\mathcal{T} = \text{Merge}(\Lambda)$ ; coalitions in  $\mathcal{T}$  decide to merge based on the merge rule explained in Section VI-B.
    - Obtain the optimal price vector  $\mathbf{p}^*$  according Algorithm 2 with input  $\mathcal{T}$ .
    - $\Lambda = \text{Split}(\mathcal{T})$ ; coalitions in  $\mathcal{T}$  decide to merge based on the Belief Pareto Order.
  - 4: **end loop** when a Bayesian-Nash stable coalitional structure  $\Lambda$  is obtained.
- 

previous single coalition  $\mathcal{T}$  according to the Belief Pareto Order; i.e., where  $\mathcal{S} \triangleright_{\text{BPO}} \mathcal{T}$ , therefore,  $\mathcal{T} \rightarrow \mathcal{S}$ .

We obtain a Bayesian-Nash stable coalitional structure of  $\mathcal{V}$ , which is an analog of the traditional core concept for the non-Bayesian coalition formation scenario.

*Definition 7:* A coalitional structure  $\Lambda = \{\Lambda_1, \dots, \Lambda_I\}$  is Bayesian-Nash stable, if no group players has an incentive to deviate based on their expected optimal payoff, even if they consider that external players react in such a way as to maximizing the payoff of deviators.

Note that both rules use  $\triangleright_{\text{BPO}}$  comparison relation 'globally,' not only focusing on the coalitions that take part and result from the merge, respectively split, but the other coalitions which are competitors, formed by the remaining MVNOs. Finally, we present a distributed coalition formation algorithm, shown in Algorithm 3 based on the Belief Pareto Order (BPO) for pricing competition and cooperation among MVNOs. The main idea is as follows. Given the initial coalition structure and coalition prices, all coalitions update their coalition prices according the process in Algorithm 2 until no coalition changes its price. Then any coalitions can merge or split, if the new one can give them more payoffs based on their beliefs. The iteration continues until no coalition changes the coalition structure.

## VII. SIMULATION RESULTS

In this section, we demonstrate the performance of our proposed pricing algorithms and cooperative strategies. We compare the pricing decisions and revenues by simulations in MATLAB under non-cooperation and cooperation cases, and also under complete information and incomplete information cases. The considered system parameters are shown in Table II, we consider a wireless virtualization communication service market which includes  $N = 2$  MNOs,  $K = 5$  MVNOs and  $L = 100$  thousand EUs with a three-stage interaction. The leased spectrum inventories for MVNOs have been determined during

TABLE II  
SIMULATION PARAMETERS

Parameters	Values	Parameters	Values
Number of MNOs $N$	2	Cost function $C_{v_k}$	$1.5 * q_k + 8$ [35]
Number of MVNOs $K$	5	Type space $\mathbb{Q}$	$\{0.7L, 1.4L, 2.1L\}$
Number of EUs $L$	100000	common prior $B$	discrete uniform distribution of $\mathbf{q}$ [26]
Reservation utility $U_0$	0	Inventory of each MVNO $q_k$	sampled uniformly from type space

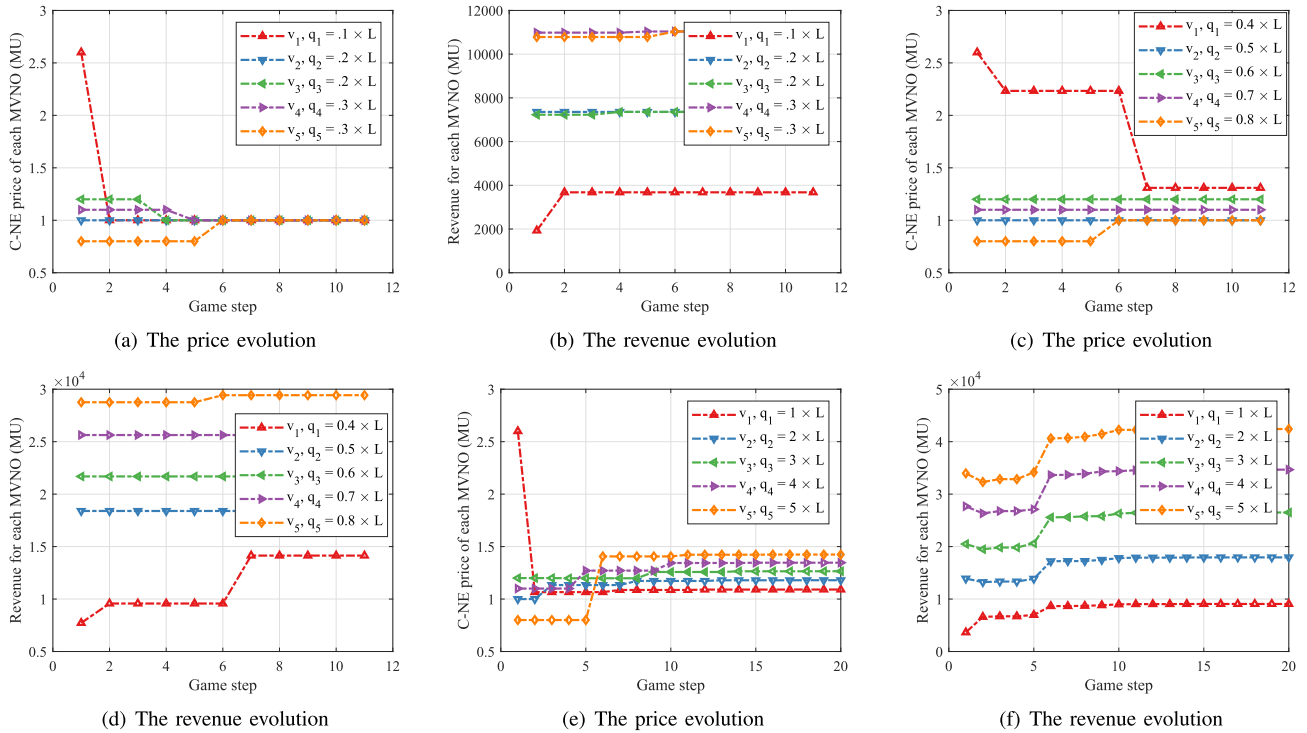


Fig. 2. The prices and revenues evolution of the MVNOs and the relationship between the price and revenue for each MVNO. For (abc), the inventories for each MVNO are  $0.1L$ ,  $0.2L$ ,  $0.2L$ ,  $0.3L$ , and  $0.3L$ , respectively. For (def), the inventories for each MVNO are  $0.4L$ ,  $0.5L$ ,  $0.6L$ ,  $0.7L$ , and  $0.8L$ , respectively. For (ghi), the inventories for each MVNO are  $1L$ ,  $2L$ ,  $3L$ ,  $4L$ , and  $5L$ , respectively.

Stage I, MVNOs make pricing decisions during Stage II, and each end user select on of MVNOs or MNOs during Stage III. We consider the stationary system state which means every user makes a choice and the choice will not change again. The inventory values will vary depending on the performance we are comparing. As for incomplete information case, we consider only three sampled values for type space, i.e.,  $\{0.7L, 1.4L, 2.1L\}$ , and all MVNOs have a common prior about the type space, which is a discrete uniform distribution.

We first consider the pricing strategy for MVNOs in non-cooperative competition with known others' spectrum inventories. Relative to the number of users, three different levels of amounts of inventories for MVNOs, i.e., small, medium, and large, are simulated. In contrast, we assume that the total amount of inventories for MNOs is equal to half the number of users. The proposed optimal price setting algorithm based on ordinal potential game is implemented. When MVNOs have a small amount of inventories, as shown in Fig. 2(a), each continuously adjusts its price in response to its rivals' price to maximize its revenue. After six iterative game step, every MVNO set the price to 1 MU. As shown in Fig. 2(b), the MVNOs that have higher

inventory obtain more revenue. In this situation, for MVNOs the competition pressure from MNOs is greater than the competition from other MVNOs. When MVNOs own a medium amount of inventories, as shown in Fig. 2(c), each MVNO set higher price than that in a small amount situation and the MVNOs that have higher inventory set higher price than lower ones. Similar to the small situation, Fig. 2(d), the MVNOs that have higher inventory obtain more revenue. In this situation, the competition MVNOs face is mainly from other MVNOs. Besides, end users get equal utility in both two situations. When MVNOs have large amount of inventories, as shown in Fig. 2(e-f), the simulation results have similar characteristics with the previous two situations except that end users can get a higher utility.

Next, we compare three kinds of cooperation strategies, proposed in Section V, i.e., proportional segmentation (PS), logarithmic segmentation (LS) and equivalent segmentation (ES), where each MVNO's inventory is also known to others. As shown in Fig. 3, the PS strategy achieves the highest sum revenue than those of the other two cooperation strategies, and the LS strategy achieves higher sum revenue than that of the ES. In addition, the difference of revenue ratio for each MVNO between

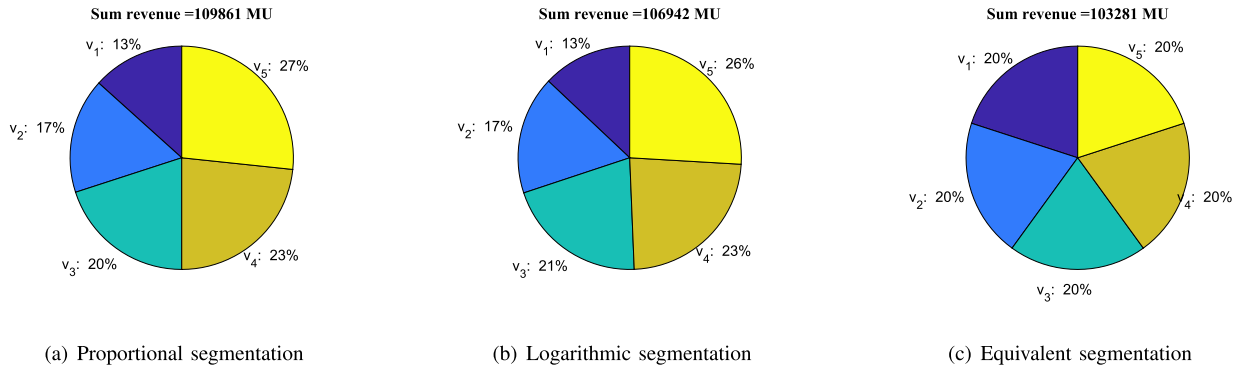


Fig. 3. The total revenue and payoff for each MVNO as to three cooperation strategies, i.e., proportional segmentation (PS), logarithmic segmentation (LS) and equivalent segmentation (ES), respectively. The inventories for each MVNO are 0.4 L, 0.5 L, 0.6 L, 0.7 L, and 0.8 L, respectively.

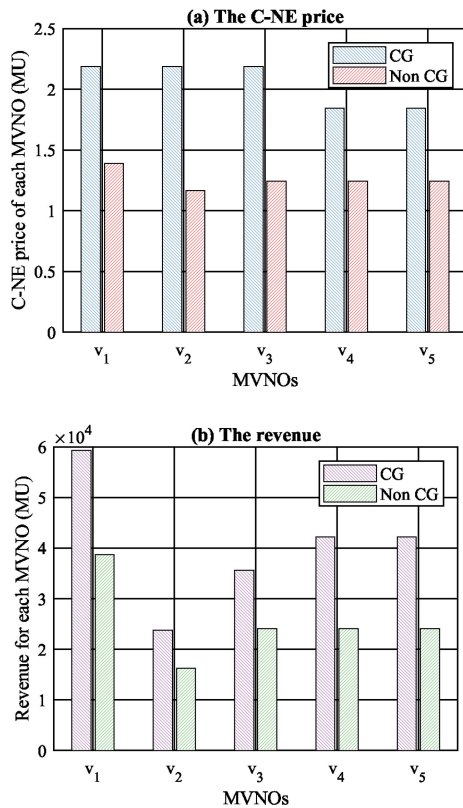


Fig. 4. The C-NE prices and revenues for MVNOs as to non-cooperative game (Non CG) and cooperative game (CG), respectively. The inventories for each MVNO are 3.5 L, 1.4 L, 2.1 L, 2.1 L, and 2.1 L, respectively. For cooperative game, the coalition structure is  $\{\{1, 2, 3\}, \{4, 5\}\}$ .

the PS and the LS is slight. For the ES, each MVNO has the same revenue with others. Furthermore, we compare the PS with the non-cooperative competition items of the price and revenue. As shown in Fig. 4, when MVNOs cooperate with others, the MVNOs can set higher price than the non-cooperation situation. Meanwhile, the revenues MVNOs obtain are improved.

Next, we consider the pricing strategy for MVNOs in cooperative competition under the condition that inventory of each MVNO is unknown to others. The inventory of each MVNO is

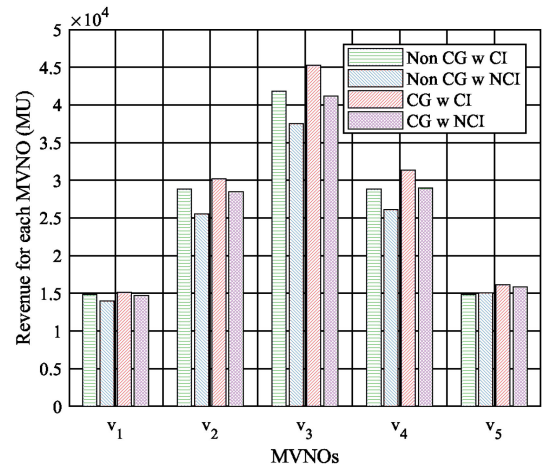


Fig. 5. The revenues for MVNOs as to cooperative game with unknown others' inventories (CG w NCI), cooperative game with known others' inventories (CG w CI), non-cooperative game with known others' inventories (Non CG w CI) and non-cooperative game with unknown others' inventories (Non CG w NCI), respectively. The true inventories for each MVNO are 0.7 L, 1.4 L, 2.1 L, 1.4 L, and 0.7 L, respectively. The common prior for all MVNOs is a discrete uniform distribution.

sampled from a discrete uniform distribution and MVNOs know only a common distribution about others' private inventories. We implement the proposed optimal price setting algorithm based on MMSE and the belief Pareto order based distributed coalition formation algorithm to achieve a Bayesian-Nash stable solution for pricing strategy. We compare the revenues from the cooperation strategy with unknown others' inventory with the revenue from cooperation strategy with known others' inventory, the revenue from non-cooperation strategy with known others' inventory, and the revenue from non-cooperation strategy with unknown others' inventory. As shown in Fig. 5, we have the following observations:

- The price decisions for MVNOs with less information about rivals may lead to lower revenues for all users including cooperation and non-cooperation competition.
- The cooperation in incomplete information may not bring more benefits than the non-cooperation when full information is available.

- To make more profits, MVNOs will get as much information about the rivals as possible, and the cooperation with full information allows MVNOs to obtain the most revenue.

### VIII. CONCLUSION

In this paper, we studied the pricing decisions for MVNOs with known or unknown others' inventory information. For the situation where the inventory information of one MVNO is known to others, we proposed an optimal price setting algorithm based on ordinal potential game to maximize perspective revenue for each MVNO. Then, we put forward three cooperation strategies for MVNOs and revealed the impacts of coalition structure on pricing decision. For the unknown inventory information situation, we proposed an optimal price setting algorithm based on MMSE to resolve the conflicts resulting from the uncertainty and developed a distributed coalition formation algorithm to achieve a Bayesian-Nash stable coalition structure that enables each MVNO to maximize respective revenue. Overall, our work provides guidance on its pricing strategies for MVNOs.

### APPENDIX

#### A. Proof of Axiom 1

For Case I, since  $U_i = U_k > 0, \forall v_i, v_k$ , one has

$$\log \frac{q_k}{L_k} - p_k = \log \frac{q_i}{L_i} - p_i,$$

and then,

$$\frac{L_i}{L_k} = \frac{e^{p_k}}{q_k} q_i e^{-p_i}.$$

Summing up  $i$  over all MVNOs  $v_i \in \mathcal{V}$  on both sides of the equation, we have

$$\frac{1}{L_k} \sum_{v_i \in \mathcal{V}} L_i = \frac{e^{p_k}}{q_k} \sum_{v_i \in \mathcal{V}} q_i e^{-p_i}.$$

Due to  $L_0 = 0$ , then  $\sum_{v_i \in \mathcal{V}} L_i = L - L_0 = L$ . Hence, we can obtain

$$L_k = \frac{L q_k e^{-p_k}}{\sum_{v_i \in \mathcal{V}} q_i e^{-p_i}}.$$

For Cases II and III,  $U_k = 0, \forall v_k \in \mathcal{V}$ , we have

$$\log \frac{q_k}{L_k} - p_k = 0,$$

and thus,

$$L_k = \frac{q_k e^{-p_k}}{L}.$$

Moreover, for Case II, it is already true that  $L_0 = 0$ ; and for Case III, it is easy to find  $L_0 = L - \sum_{v_i \in \mathcal{V}} q_k e^{-p_k}$ .

From the above, we prove the Axiom 1.

#### B. Proof of Lemma 1

Since the continuity and quasi-concave, the maximum value of  $R_k$  is obtained at the point where its derivative is zero. For

Case I, denoting the best response price as  $\mu_k$  for MVNO  $v_k$ , according to (8), one has

$$\frac{q_k L \left( q_k - (\mu_k - 1) e^{\mu_k} \sum_{v_i \in \mathcal{V}/v_k} q_i e^{-p_i} \right)}{\left( e^{\mu_k} \sum_{v_i \in \mathcal{V}} q_i e^{-p_i} \right)^2} = 0.$$

It is obvious that this equation holds if and only if the numerator is zero, and then the following equation holds,

$$q_k - (\mu_k - 1) e^{\mu_k} \sum_{v_i \in \mathcal{V}/v_k} q_i e^{-p_i} = 0.$$

Rearranging the terms, one can obtain

$$(\mu_k - 1) e^{\mu_k} = \frac{q_k}{\sum_{v_i \in \mathcal{V}/v_k} q_i e^{-p_i}}.$$

For Case II, according to (8), one has

$$\begin{cases} \log \frac{q_i}{L_i} - p_i = 0, \forall v_i \in \mathcal{V}, \\ L_k + \sum_{v_i \in \mathcal{V}/v_k} L_i = L. \end{cases} \quad (28)$$

Thus, we can obtain

$$p_k^* = \eta_k = \log \frac{q_k}{L - \sum_{v_i \in \mathcal{V}/v_k} q_i e^{-p_i}} \quad (29)$$

by solving the above equation array corresponding the camber condition with Case II in Section III-B.

For Case III, we have

$$\frac{\partial R_k}{\partial p_k} = q_k (1 - p_k) e^{-p_k} = 0,$$

and hence,  $p_k^* = 1$ .

Based on the above, Lemma 1 is proved.

#### C. Proof of Theorem 1

For Case I, according to (8), we have

$$p_k^* < \log \frac{q_k}{L - \sum_{v_i \in \mathcal{V}/v_k} q_i e^{-p_i^*}},$$

and thus,

$$\sum_{v_i \in \mathcal{V}} q_i e^{-p_i^*} > L.$$

According to Lemma 1, we can get (15). Then substituting the  $k$ -th item in (15) into Case I of (8), we can obtain the following equation,

$$R_k^* = L(p_k^* - 1).$$

For the other two cases, we can draw the corresponding conclusions, by applying a similar derivation process.

#### D. Proof of Proposition 1

Without loss of generality, we assume MVNO  $v_k$  changes its inventory, and observe the price dynamics of itself and any other

one MVNO  $v_i, \forall v_i \in \mathcal{V}$ . According to (15), one has

$$\begin{cases} (p_i^* - 1)e^{p_i^*} = \frac{q_i}{q_k e^{-p_k^*} + \sum_{v_j \in \mathcal{V}/\{v_i, v_k\}} q_j e^{-p_j^*}}, \\ (p_k^* - 1)e^{p_k^*} = \frac{q_k}{q_i e^{-p_i^*} + \sum_{v_j \in \mathcal{V}/\{v_i, v_k\}} q_j e^{-p_j^*}}. \end{cases} \quad (30)$$

By arranging the items and removing the summation item, we can get

$$q_i \frac{p_i^*}{(p_i^* - 1)} e^{-p_i^*} = q_k \frac{p_k^*}{(p_k^* - 1)} e^{-p_k^*}. \quad (31)$$

Defining  $g(p)$  as the following function of the variable  $p$ ,

$$g(p) = \frac{p-1}{pe^{-p}}, \quad (32)$$

its derivative is

$$\frac{\partial g(p)}{\partial p} = \frac{(p - \frac{1}{2})^2 + \frac{3}{4}}{p^2} > 0. \quad (33)$$

This means that  $g(p)$  is a monotonically increasing function about  $p$ , and therefore we have

$$\frac{g(p^{r_k^*})}{g(p^{r_i^*})} = \frac{q_k}{q_i}. \quad (34)$$

When  $q_k$  increases with constant  $q_i$ ,  $g(p^{r_k^*})$  increases accordingly, and thus  $p^{r_k^*}$  increases. In contrast,  $g(p^{r_i^*})$  decreases accordingly, and thus  $p^{r_i^*}$  decreases.

### E. Proof of Proposition 2

Since  $q_i = q_j$ , according to (31), we have

$$\frac{p_i^*}{(p_i^* - 1)} e^{-p_i^*} = \frac{p_j^*}{(p_j^* - 1)} e^{-p_j^*}. \quad (35)$$

Meanwhile, due to the monotonic decrease of function  $g(p)$  according to (33), only and if only  $p_i^* = p_j^*$ , (35) holds.

### F. Proof of Theorem 2

Since the maximum value is obtained at a zero derivative, for MVNO  $v_k$  to maximize its revenue, one has

$$\frac{\partial R_{\Lambda_n}(p_{\Lambda_n})}{\partial p_{\Lambda_n}} = 0 \Rightarrow e^{p_{\Lambda_n}}(p_{\Lambda_n} - 1) = \frac{\mathbf{q}_{\Lambda_n}}{\sum_{v_i \in \mathcal{V}/\Lambda_n} q_i e^{-p_i}} \quad (36)$$

In addition, to maximize the overall revenue of all the member MVNOs, the coalition price can be calculated by

$$\frac{\partial R_k(p_{\Lambda_n})}{\partial p_{\Lambda_n}} = 0 \Rightarrow e^{p_{\Lambda_n}}(p_{\Lambda_n} - 1) = \frac{\mathbf{q}_{\Lambda_n}}{\sum_{v_i \in \mathcal{V}/\Lambda_n} q_i e^{-p_i}} \quad (37)$$

Therefore, the coalition set from coalition perspective is same with the one from individual perspective.

### G. Proof of Theorem 3

Let  $R_{\Lambda_n}$  denote aggregate revenue of coalition  $\Lambda_n$ . When the Cournot NE is in Case I, it is given as:

$$R_{\Lambda_n}^* = \frac{\mathbf{q}_{\Lambda_n} p_{\Lambda_n}^* L}{\mathbf{q}_{\Lambda_n} + e^{p_{\Lambda_n}^*} \sum_{v_i \in \mathcal{V}/\Lambda_n} q_i e^{-p_i^*}}. \quad (38)$$

NE price of each coalition satisfies the following equation:

$$\begin{aligned} \frac{\partial R_{\Lambda_n}^*(p_{\Lambda_n}^*, \mathbf{p}_{-\Lambda_n}^*, \mathbf{q}_{\Lambda_n}, \mathbf{q}_{-\Lambda_n})}{\partial p_{\Lambda_n}^*} &= 0 \\ \Rightarrow e^{p_{\Lambda_n}^*} (p_{\Lambda_n}^* - 1) &= \frac{\mathbf{q}_{\Lambda_n}}{\sum_{v_i \in \mathcal{V}/\Lambda_n} q_i e^{-p_i^*}} \end{aligned} \quad (39)$$

Using (38) and (39), we can obtain

$$R_{\Lambda_n}^* = L(p_{\Lambda_n}^* - 1). \quad (40)$$

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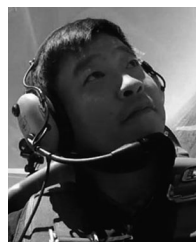
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