Hedonic Coalition Formation Games for Secondary Base Station Cooperation in Cognitive Radio Networks

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Abstract— **In order to maintain a conflict-free environment among licensed primary users (PUs) and unlicensed secondary users (SUs) in cognitive radio networks, providing frequency and geographical information through control channels, such as the cognitive pilot channel (CPC), has been recently proposed. While existing literature focused on the type of information that these control channels need to carry, this paper investigates the problem of gathering this information cooperatively, among a network of secondary base stations (SBSs). In this regard, given a cognitive network where every SBS can only have accurate knowledge on a small number of different primary users (PUs) or channels, each SBS can cooperate with neighboring SBSs in order to improve its view of the spectrum, i.e., learn about new PUs that can subsequently be used by its served SUs. We model the problem as a hedonic coalition formation game among the SBSs and we propose an algorithm for forming the coalitions. Using the proposed algorithm, each SBS can take an individual decision to join or leave a coalition while maximizing its overall potential utility, which accounts for the tradeoff between the benefit from learning new channels through coalition members and the cost from receiving inaccurate information. Simulation results show that the proposed algorithm yields a performance advantage, in terms of the average payoff per SBS reaching up to 165% relative to the non-cooperative case for a large network with 27 SBSs.**

I. Introduction

Recently, the telecommunications world witnessed a surge in wireless services which has significantly increased the demand for the radio spectrum. However, the radio spectrum resources are scarce and have, in majority, been already licensed to existing operators. Recently, it has been shown that the actual licensed spectrum remains unoccupied for large periods. Thus, *cognitive radio* (CR) systems have been proposed [1] in order to efficiently exploit these *spectrum holes* during which the licensed primary users (PUs) are inactive. CRs or secondary users (SUs) are wireless devices that can learn from their environment and, hence, they are able to share the spectrum with the PUs, operating whenever the PUs are idle. Implementing CR systems is a challenging task due to the need for a conflict-free coexistence between the SUs and the PUs [2].

Recently, it has been established [3] (and the references therein) that, in practice, the sensing time of CR networks, i.e., the time during which the SUs learn their environment, is *large* and affects the access performance of the SUs. For this reason as well as due to the increasing amount of information needed for efficient coexistence between the SUs and the PUs, there has been a recent interest in providing, through control channels, assistance to the SUs on both the spectrum sensing and access levels in order to improve the SUs' sensing time, their access strategies, as well as their energy consumption [4–8]. In [4], a control channel, called the Common Spectrum Coordination Channel (CSCC) has been proposed for the announcement of radio and service parameters to allow a coordinated coexistence between SUs and PUs. Following a similar concept, the Cognitive Pilot Channel (CPC) has been recently proposed [5–8] as a means for providing frequency and geographical information to cognitive users, assisting them in sensing and accessing the spectrum, notably in the presence of multiple access technologies and PUs. As explained in [5–7], the CPC is a control channel that carries information such as available spectrum opportunities, existing frequencies, i.e., PUs or geographical information, allowing the SUs to improve their performance and avoid scanning the entire spectrum for identifying spectrum holes and available PUs. The different implementation possibilities for deploying the CPC in cognitive networks are tackled in [5–8]. Namely, in [8], a broadcast and an on-demand method for delivering the CPC data to the SUs have been proposed. Further, the various information that the CPC can carry are also investigated and detailed in [5–8].

The deployment of control channels such as the CPC, for assisting the SUs in their spectrum exploration and exploitation is a key technology for next generation cognitive systems. The main existing work in this area focused on the information that these channels can convey, as well as on implementation aspects. To the best of our knowledge, no work has investigated how a number of secondary cognitive base stations (SBSs) can interact to share their knowledge on available PUs' in order to improve the quality and amount of information carried over their control channel, hence, improving the performance of their served SUs.

The main contribution of this paper is to provide a cooperative scheme among the SBSs in a multi-channel cognitive network which allows them to share their knowledge on existing PUs and their availability, and, hence, improve the quality of the information conveyed over their control channels, e.g., the CPC. By cooperating and learning the existence of alternative PUs (channels) and their state, the SBSs can offer a better service to their served SUs. We model the problem as a hedonic coalition formation game and we propose an algorithm for forming the SBS coalitions. Using the proposed algorithm, every SBS can take an individual decision to join or leave a coalition, while maximizing its overall potential utility which accounts for the gains resulting from the exploration of the state of new PU channels, as well as the costs resulting from receiving inaccurate information on some channels from some of the cooperating partners. Thus, the proposed algorithm allows the SBSs to self-organize into disjoint coalitions that constitute a Nash-stable network partition. Within every formed coalition, the SBSs build their control channels cooperatively by sharing their view of the spectrum over the set of PUs that they are aware of and able to sense. Simulation results show that the proposed algorithm

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presents a significant performance advantage, in terms of average payoff per SBS, reaching up to 165% improvement relative to the non-cooperative case.

The rest of this paper is organized as follows: Section II presents the system model. In Section III, we present the proposed cooperation protocol and model it using hedonic coalition formation games. In Section IV, we propose an algorithm for coalition formation. Simulation results are analyzed in Section V and conclusions are drawn in Section VI.

II. System Model

Consider a cognitive network with N secondary base stations (SBSs) with each SBS i serving a number L_i of secondary users (SUs) in a specific geographical area, i.e., each SBS provides coverage for a specific cell or mesh. We denote by $\mathcal{N}$ the set of all SBSs. Within this network, K primary users (PUs) exist with the set of PUs denoted by $\mathcal{K}$. Each PU consists of a transmitter-receiver pair that operates over a certain wireless channel. Whenever any PU $k \in \mathcal{K}$ is not transmitting, the SUs can use PU k's channel in order to transmit their information. Hereafter, we use the terms PU and channel interchangeably, as every PU corresponds to a certain channel. We denote by θ_k as the probability that PU $k \in \mathcal{K}$ is *not* transmitting, and, hence, channel k is *available*. Each SBS $i \in \mathcal{N}$ can only gather accurate statistics regarding a subset $\mathcal{K}_i \subseteq \mathcal{K}$ of $K_i \leq K$ PUs, during the period of time the channels remain stationary [1], [2]. Subsequently, using a control channel such as the cognitive pilot channel (CPC), each SBS can convey the statistics and availability of these PUs to their served SUs in order to assist them in their spectrum exploration and exploitation. Using this information, the SUs can decide on which channels to access.

For each SBS $i \in \mathcal{N}$, in order to detect the PUs in $\mathcal{K}_i$, the SBS uses energy detectors which are one of the main practical signal detectors in cognitive radio networks [9]. In such a non-cooperative setting, assuming Rayleigh fading, for every SBS $i \in \mathcal{N}$, the probability that SBS i accurately detects the presence of any PU $k \in \mathcal{K}_i$ is given by [9, Eq. (4)]

$$P^i_{\text{det},k} = e^{-\frac{\lambda_{i,k}}{2}} \sum_{n=0}^{m-2} \frac{1}{n!}\left(\frac{\lambda_{i,k}}{2}\right)^n + \left(\frac{1+\bar{\gamma}_{ki}}{\bar{\gamma}_{ki}}\right)^{m-1} \times \left[e^{-\frac{\lambda_{i,k}}{2(1+\bar{\gamma}_{ki})}} - e^{-\frac{\lambda_{i,k}}{2}} \sum_{n=0}^{m-2} \frac{1}{n!}\left(\frac{\lambda_{i,k}\bar{\gamma}_{ki}}{2(1+\bar{\gamma}_{ki})}\right)^n\right], \quad (1)$$

where m is the time bandwidth product, $\lambda_{i,k}$ is the energy detection threshold selected by SBS i for sensing channel k. $\bar{\gamma}_{ki}$ represents the average SNR of the received signal from PU $k \in \mathcal{K}_i$ to SBS i and is given by $\bar{\gamma}_{ki} = \frac{P_k g_{ki}}{\sigma^2}$ with P_k the transmit power of PU k, σ^2 the Gaussian noise variance, $g_{ki} = 1/d^{\mu}_{ki}$ the path loss between PU k and SBS i, μ the path loss exponent, and d_{ki} the distance between PU k and SBS i.

Further, the false alarm probability perceived by SBS $i \in \mathcal{N}$ over PU channel $k \in \mathcal{K}_i$ is given by [9, Eq. (2)]

$$P^i_{\text{fal},k} = \frac{\Gamma(m, \frac{\lambda_{i,k}}{2})}{\Gamma(m)}, \quad (2)$$

where $\Gamma(\cdot,\cdot)$ is the incomplete gamma function and $\Gamma(\cdot)$ is the gamma function.

Consequently, for every SBS $i \in \mathcal{N}$, in a non-cooperative manner, the total *potential* utility that SBS i can obtain through the information it conveys to its served SUs over its control channel, e.g., its CPC can be given by

$$v(\{i\}) = \sum_{k \in \mathcal{K}_i} \sum_{j=1}^{L_i} \Big[(1-P^i_{\text{fal},k})\theta_k \rho_{ji} - \alpha_k (1-P^i_{\text{det},k})(1-\theta_k)(\rho_{kr_k} - \rho^j_{kr_k})\Big] \quad (3)$$

with L_i the number of SUs served by SBS i and α_k a penalty factor imposed by PU k for any SU that causes interference on k. The term $(1-P^i_{\text{det},k})$ represents the probability of miss, i.e., the probability that SBS i considers channel k is available while the PU is actually transmitting. Further, ρ_{ji} represents the probability that, during the transmission of SU j to its serving SBS i at the time channel k is available, the received SNR at SBS i is higher than a target desired level ν_0 (probability of successful transmission), and is given by [10] (Rayleigh fading) $\rho_{ji} = \exp\left(\frac{-\nu_0}{\bar{\gamma}_{ji}}\right)$, where $\bar{\gamma}_{ji} = \frac{P_j g_{ji}}{\sigma^2}$ is the average SNR received at SBS i from SU j with P_j the transmit power of SU j and $g_{ji} = \frac{1}{d^{\mu}_{ji}}$ the channel gain between SU j and SBS i. The target SNR ν_0 is assumed the same for all SBSs, SUs and PUs.

Further, $(\rho_{kr_k} - \rho^j_{kr_k})$ represents the reduction in the probability of successful transmission $\rho_{kr_k} = \exp\left(\frac{-\nu_0}{\bar{\gamma}_{kr_k}}\right)$ ($\bar{\gamma}_{kr_k}$ is the average SNR achieved by PU k at its receiver r_k) of PU k at its receiver r_k, whenever SU j transmits over channel k at a time when PU k is present (due to missing the detection of PU k). In this context, the probability $\rho^j_{kr_k} = \exp\left(\frac{-\nu_0}{\bar{\gamma}^j_{kr_k}}\right)$ with $\bar{\gamma}^j_{kr_k} = \frac{P_k g_{kr_k}}{\sigma^2 + g_{jk}P_j}$ represents PU k's probability of successful transmission at its receiver r_k when SU j is transmitting over channel k during the time this channel is occupied by PU k (g_{jk} is the channel gain between SU j and PU k). Consequently, the utility in (3) represents the total potential utility that any SBS $i \in \mathcal{N}$ achieves through the information it provides over its control channel/CPC to its served SUs. Every term in the double summation of (3) is the difference between the performance achieved when correct detection decisions are made by SBS i, and the penalty imposed by the PU on the SBS when one of its SUs accesses a channel while the PU is using that channel, i.e., due to the probability of miss of the SBS.

For improving their utilities in (3), the SBSs can cooperate and share their knowledge of the existing PUs. By cooperating, the SBSs can learn the availability of additional PUs and, hence, become able to provide a more diverse channel selection for their SUs. In the following section, we discuss the proposed cooperative protocol among the SBSs and formulate it as a hedonic coalition formation game.

III. Secondary Base Station Cooperation Game

As every SBS is able to learn the statistics and provide the information on only a subset $\mathcal{K}_i \subseteq \mathcal{K}$ of PUs, there is a need for some SBSs to cooperate and share their knowledge of the spectrum, in order to improve the information they convey using their control channels. However, while sharing the availability of certain channels, some information may be received *inaccurately* by certain SBSs, for example, due to the different channel conditions that the cooperating SBSs perceive about some PUs. In this regard, there exists an inherent cooperative tradeoff between the benefits gained from learning new channels and the costs in terms of the accuracy of the information provided through cooperation, i.e., the trust between the cooperating SBSs.

Whenever any given coalition $S \subseteq \mathcal{N}$ of SBSs forms, its members can continuously share the statistics and information on their known PUs. Hence, the total number of known PUs by any SBS i member of coalition S, i.e., $i \in S$ is $\mathcal{K}_S = \cup_{j\in S}\mathcal{K}_j$ where $\mathcal{K}_j \subseteq \mathcal{K}$ is the set of PUs known by any SBS $j \in S$. Further, given any SBS $i \in S$, for any channel $k \in \mathcal{K}_S \setminus \mathcal{K}_i$, SBS i has to learn the channel availability θ_k from one of the other members of S. Consequently, given a coalition S and any SBS $i \in S$, the perceived availability $\bar{\theta}^k_{ij}$ of any channel $k \in \mathcal{K}_S \setminus \mathcal{K}_i$ learned by SBS i through another SBS $j \in S$, is given by

$$\bar{\theta}^k_{ij} = \tau^k_{ij}\theta_k, \tag{4}$$

where $0 \leq \tau^k_{ij} \leq 1$ represents the level of trust that SBS i has for the information provided by SBS j on the availability of PU k. Note that, $\tau^k_{ii} = 1, \forall i \in \mathcal{N}, k \in \mathcal{K}_i$. Ultimately, denoting by H^k_0 the state in which PU k is idle, i.e., not transmitting, the trust value τ^k_{ij} perceived by any SBS $i \in \mathcal{N}$ about another SBS $j \in \mathcal{N}$ from which SBS i is aiming to learn the statistics of PU $k \in \mathcal{K}$ converges to the following probability:

$$\tau^k_{ij} = \Pr(i = H^k_0 | j = H^k_0) = \frac{\Pr(i = H^k_0, j = H^k_0)}{\Pr(j = H^k_0)}, \tag{5}$$

where $\Pr(i = H^k_0 | j = H^k_0)$ is the probability that SBS i perceives channel k as available given that SBS j claims so. Further, $\Pr(i = H^k_0, j = H^k_0) = (1 - P^i_{\text{fal},k})(1 - P^j_{\text{fal},k})\theta_k + (1 - P^i_{\text{det},k})(1 - P^j_{\text{det},k})(1 - \theta_k)$ where $P^i_{\text{det},k}, P^j_{\text{det},k}$ are, respectively, the probabilities of detecting PU k by SBSs i and j as given by (1) and $P^i_{\text{fal},k}, P^j_{\text{fal},k}$ are, respectively, the false alarm probabilities for SBSs i and j over channel k as given by (2). Further, we have $\Pr(j = H^k_0) = (1 - P^j_{\text{fal},k})\theta_k + (1 - P^j_{\text{det},k})(1 - \theta_k)$. In practice, SBS i is unaware of the statistics of PU k and needs to learn these statistics from SBS j. Hence, SBS i cannot use the expression in (5) to compute the trust value, since it is unaware of its own probability of detection $P^i_{\text{det},k}$ for channel k. Subsequently, for estimating these trust values, prior to any formation of coalitions, the SBSs interact and, by using standard trust learning algorithms (e.g. in [11]), they can compute an estimate for the trust values, which they will subsequently utilize to decide on whom to cooperate with.

Consequently, given any coalition $S \subseteq \mathcal{N}$, the payoff $\phi_i(S)$ of any SBS $i \in S$ is given by

$$\phi_i(S) = \sum_{k\in\mathcal{K}_S}\sum_{j=1}^{L_i}\Big[(1 - P^{i_k}_{\text{fal},k})\bar{\theta}^k_{ii_k}\rho_{ji} - \alpha_k(1 - P^{i_k}_{\text{det},k})(1 - \bar{\theta}^k_{ii_k})(\rho_{kr_k} - \rho^j_{kr_k})\Big] \tag{6}$$

where $\bar{\theta}^k_{ii_k}$ is given by (4), and $i_k \in S$ indicates the SBS in S that is providing the information on channel k for SBS i. For any channel that SBS i is able to sense on its own, i.e., for any $k \in \mathcal{K}_i$ we have $i_k = i$, however, for any channel that SBS i needs to learn from others, i.e., for any $k \in \mathcal{K}_S \setminus \mathcal{K}_i$, we have $i_k = \arg\max_{j\in S\setminus\{i\}}(\sum_{j=1}^{L_i}((1 - P^{i_k}_{\text{fal},k})\bar{\theta}_k\rho_{ji} - \alpha_k(1 - P^{i_k}_{\text{det},k})(1 - \bar{\theta}_k)(\rho_{kr_k} - \rho^j_{kr_k})))$. In other words, any SBS i part of a coalition S relies on its own sensing results for any channel that it already knows, i.e. channels in $\mathcal{K}_i$, while for any channel $k \in \mathcal{K}_S \setminus \mathcal{K}_i$ that it needs to learns from its cooperating partners in $S \setminus \{i\}$, SBS i selects the partner which gives the maximum benefit.

For mathematically modeling the SBS cooperation problem, we refer to coalitional game theory [12], [13]. Thus, the introduced SBSs cooperation problem can be modeled as a coalitional game with a non-transferable utility [12, Chap. 9]:

Definition 1: A coalitional game with *non-transferable utility* is defined by a pair $(\mathcal{N}, V)$ where $\mathcal{N}$ is the set of players and V is a mapping such that for every coalition $S \subseteq \mathcal{N}$, $V(S)$ is a closed convex subset of $\mathbb{R}^S$ that contains the payoff vectors that players in S can achieve.

For the proposed game, the mapping V is defined as

$$V(S) = \{\boldsymbol{x}(S) \in \mathbb{R}^S | \;\; x_i(S) = \phi_i(S),\; \forall i \in S\}, \tag{7}$$

where $\phi_i(S)$ is given by (6). The proposed game is an $(\mathcal{N}, V)$ coalitional game with non-transferable utility since the mapping V as per (6) and (7) is a singleton set, hence, closed and convex.

Furthermore, as clearly seen from (4) and (6), due to the trust between the SBSs, cooperation entails costs, and, hence, is not always beneficial for the SBSs. Due to the trust costs, the size of a coalition is limited and certain SBSs may never cooperate. Hence, traditional solution concepts for coalitional games, such as the core [12], may not be applicable [13], [14]. In fact, in order for the core to exist as a solution concept, a coalitional game must ensure that the grand coalition, i.e., the coalition of all players will form. For the proposed game, as corroborated by (4) and (6), in general, the grand coalition will not always form due to the costs for cooperation. Instead, independent and disjoint SBS coalitions will appear in the network. In this regard, the proposed SBSs cooperation game is classified as a *coalition formation game* [13], [14], and the objective is to find an algorithm that allows to form the SBS coalitions.

For instance, the proposed SBS coalitional game can be modeled using a class of coalition formation games known as *hedonic coalition formation* games [13], [15]. Notably, this class of games entails several interesting properties that can be applied, not only in economics such as in [15] (and references therein), but also in wireless networks' problems such as the proposed SBSs game. A coalition formation game is classified as *hedonic* if and only if: (i)- The payoff of any player depends *solely* on the members of the coalition to which the player belongs, and (ii)- The coalitions form as a result of the *preferences* of the players over their possible coalitions' set. In the proposed SBS game, by inspecting (6) one can see that the payoff of any SBS i depends only on the identity of the SBSs in the coalition to which SBS i belongs, with no dependence on any other SBS. Hence, the first hedonic condition is verified. For the second condition, prior to defining how the preferences of the players over the coalitions can be used in our model, we introduce the following definition:

Definition 2: A *coalitional structure* or *coalition partition* is defined as the set $\Pi = \{S_1, \ldots, S_l\}$ which partitions the players set $\mathcal{N}$, i.e., $\forall\, k \in \{1, \ldots, l\}, S_k \subseteq \mathcal{N}$ are disjoint coalitions such that $\cup^l_{k=1}S_k = \mathcal{N}$.

In a hedonic game setting such as in the proposed SBS coalition formation game, each player can build preferences over its own set of possible coalitions using the concept of a preference relation or order as follows [15]:

Definition 3: For any player $i \in \mathcal{N}$, a *preference relation* or *order* $\succeq_i$ is defined as a complete, reflexive, and transitive binary relation over the set of all coalitions that player i can possibly form, i.e., the set $\{S_k \subseteq \mathcal{N} : i \in S_k\}$.

Consequently, for a player $i \in \mathcal{N}$, given two coalitions $S_1 \subseteq \mathcal{N}$ and, $S_2 \subseteq \mathcal{N}$ such that $i \in S_1$ and $i \in S_2$, $S_1 \succeq_i S_2$ implies that player i prefers to be member of coalition S_1, over being member of coalition S_2, or i is indifferent between S_1 and S_2. Further, using the asymmetric counterpart of $\succeq_i$, denoted by $\succ_i$, then $S_1 \succ_i S_2$, indicates that player i *strictly* prefers being a member of S_1 over being a member of S_2. For every application, an adequate preference relation $\succeq_i$ can be defined to allow the players to quantify their preferences. Given the set of players $\mathcal{N}$, and a preference relation $\succeq_i$ for every player $i \in \mathcal{N}$, a hedonic coalition formation game is defined by the pair $(\mathcal{N}, \succ)$ where $\succ$ is a *profile of preferences*, i.e., preference relations, $(\succeq_1, \ldots, \succeq_N)$ defined for every player in $\mathcal{N}$ [15].

For the proposed coalition formation game, we propose the following preference relation for any SBS $i \in \mathcal{N}$, in order to cast the game as a $(\mathcal{N}, \succ)$ hedonic coalition formation game:

$$S_1 \succeq_i S_2 \Leftrightarrow w_i(S_1) \geq w_i(S_2), \tag{8}$$

where $S_1,\ S_2 \subseteq \mathcal{N}$, are any two coalitions that contain SBS i, i.e., $i \in S_1$ and $i \in S_2$ and w_i is a preference function defined for any SBS $i \in \mathcal{N}$ and any coalition S such that $i \in S$ as follows

$$w_i(S) = \begin{cases} x_i(S), & \text{if } (x_j(S) \geq x_j(S \setminus \{i\}), \\ & \quad \forall j \in S \setminus \{i\} \&\ h_i(S) \leq \eta_i) \text{ or } (|S| = 1), \\ -\infty, & \text{otherwise,} \end{cases} \tag{9}$$

where $x_i(S)$ is the payoff received by SBS i in coalition S as per given by (6) and (7), $h_i(S)$ is the number of times that SBS i visited and then left coalition S (i.e., $h_i(S)$ reflects the history of the coalitions that i has visited), and η_i is a threshold used by SBS i for $h_i(S)$ indicating the maximum number of times that SBS i will visit and leave a certain coalition S before never revisiting S. Consequently, the proposed SBS coalitional game verifies both hedonic conditions, and, hence, we have:

Property 1: The proposed SBS coalition formation game is modeled as a $(\mathcal{N}, \succ)$ hedonic coalition formation game, with the preference relation $\succeq_i$ given by (8) for any SBS $i \in \mathcal{N}$.

IV. Hedonic Coalition Formation Algorithm

For the proposed $(\mathcal{N}, \succ)$ hedonic SBS coalition formation game, we propose the following rule for coalition formation:

Definition 4: Switch Rule - Given a partition $\Pi = \{S_1, \ldots, S_M\}$ of the SBSs' set $\mathcal{N}$, an SBS i decides to leave its current coalition S_m, for some $m \in \{1, \ldots, M\}$ and join another coalition $S_k \in \Pi \cup \{\emptyset\}$, $S_k \neq S_m$, hence forming $\Pi' = \{\Pi \setminus \{S_m, S_k\}\} \cup \{S_m \setminus \{i\}, S_k \cup \{i\}\}$, if and only if $S_k \cup \{i\} \succ_i S_m$. Hence, $\{S_m, S_k\} \rightarrow \{S_m \setminus \{i\}, S_k \cup \{i\}\}$ and $\Pi \rightarrow \Pi'$.

The switch rule provides a mechanism through which any SBS is able to take an individual decision to leave its current coalition S_m and join a different coalition $S_k \in \Pi$, as long as $S_k \cup \{i\} \succ_i S_m$ as per (8). Thus, an SBS would *switch* to a new coalition if it can strictly improve its payoff, *without* decreasing the payoff of any member of the new coalition (given the *consent* of these members) as per (8) and (9). Note that whenever an SBS leaves a coalition $S_m,\ |S_m| > 1$ to join another coalition (occurrence of a switch), the history counter $h_i(S_m)$ is increased by 1.

Subsequently, the proposed hedonic coalition formation algorithm is composed of three main stages: learning stage, hedonic coalition formation stage, and control channel transmission stage. In the first stage, prior to any coalition formation, the SBSs can discover their neighboring SBSs through algorithms such as in [16], and, by using standard trust learning algorithms (e.g. in [11]), each SBS in $\mathcal{N}$ can form an estimate of the trust values in (5) that it assigns to the other SBSs. Following the learning stage, hedonic coalition formation begins. First, each SBS starts by investigating the possibility of performing a switch operation by engaging in pairwise negotiations with discovered SBSs/coalitions. Once a potential switch operation is identified (using (8)), every SBS can make a distributed decision to switch to the newly preferred coalition. In this stage, we consider that the SBSs make their switch decisions in a random, but sequential order (dictated by the SBS who makes the first request to cooperate). Each SBS can easily make the decision to switch as it leaves its current coalition and coordinates, through the backbone, the joining of the new coalition whose members agree on the joining of this SBS as per (8) and (9). The convergence of the proposed coalition formation algorithm during this phase is guaranteed as follows:

Theorem 1: Beginning with any initial network partition Π_{init}, the hedonic coalition formation stage of the proposed algorithm always converges to a final network partition Π_f composed of a number of disjoint coalitions of SBSs.

Proof: Due to space limitation, we provide only a succinct sketch of the proof. The convergence is mainly a result of the defined preferences in (8) and (9) and the fact that he number of partitions of a set is *finite* (as given by the Bell number [14]). As per (8) and (9), one can see that, whenever an SBS $i \in \mathcal{N}$ makes a switch operation, it can only revisit a certain coalition (with size greater than 1) for a maximum of η_i times. After leaving any coalition that was revisited η_i times (if such a case occurs), an SBS i would definitely visit a new coalition. Although the SBS can also decide to revert to acting non-cooperatively at any time (switch to a coalition of size 1), once this SBS acts non-cooperatively it can either remain non-cooperative forever or, ultimately, rejoin a new coalition, hence, yielding a new partition. Hence, the proposed hedonic coalition formation phase is a sequence of switch operations where it is guaranteed that at a certain point in time each switch operation would result in a newly visited partition, and due to the finite number of partitions of a set, this sequence of switch operations (hedonic coalition formation stage) always converges to a final partition Π_f. ■

We study the stability of the partition Π_f resulting from the convergence of our algorithm using the following concept [15]:

Definition 5: A partition $\Pi = \{S_1, \ldots, S_M\}$ is *Nash-stable* if $\forall i \in \mathcal{N}$ s.t. $i \in S_m, S_m \in \Pi$, $S_m \succeq_i S_k \cup \{i\}$ for all $S_k \in \Pi \cup \{\emptyset\}$.

Hence, a partition Π is Nash-stable, if no SU has an incentive to move from its current coalition to another coalition in Π or to deviate and act alone.

Proposition 1: Any partition Π_f resulting from the coalition formation phase of the proposed algorithm is Nash-stable.

TABLE I

ONE ROUND OF THE PROPOSED SBS COOPERATION ALGORITHM

Initial State

The network is partitioned by Π_i=$\{S_1, \ldots, S_k\}$ (At the beginning of all time Π_i = $\mathcal{N} = \{1, \ldots, N\}$ with non-cooperative SBSs).

Three stages in each round of the algorithm

Stage 1 - Learning Stage:

Each individual SBS discovers its neighboring SBSs and their characteristics using algorithm such as in [16].

Each individual SBS, via trust learning algorithms (e.g. in [11]) computes an estimate of the trust values in (5).

Stage 2 - Hedonic Coalition Formation:

repeat

Each SBS $i \in \mathcal{N}$ engages in pairwise negotiations with discovered neighbors, to identify potential switch operations.

Once a switch operation is found using (8):

a) SBS i leaves its current coalition S_m and updates $h_i(S_m)$ if necessary.

b) SBS i joins a new coalition S_k with the consent of the members of S_k.

until convergence to a Nash-stable partition.

Stage 3 - Control Channel Transmission:

a) Each SBS gathers information from its coalition members.

b) The SBS builds its control channel, e.g., the CPC

c) The information is conveyed within every cell over the control channel using methods such as in [8]

Proof: If the partition Π_f resulting from the proposed algorithm is *not* Nash-stable then, there $\exists i \in \mathcal{N}$ with $i \in S_m$, $S_m \in \Pi_f$, and a coalition $S_k \in \Pi_f$ such that $S_k \cup \{i\} \succ_i S_m$. Hence, SBS i can perform a *switch* operation which contradicts with the fact that Π_f is the result of the convergence of the proposed algorithm (Theorem 1). Thus, any partition Π_f resulting from the hedonic coalition formation stage is Nash-stable. ■

Following the convergence of the hedonic coalition formation stage to a Nash-stable partition, the third and last stage of the algorithm entails the construction and transmission of the information by the SBSs. In this stage, each SBS can convey the information it collected from its coalition members, along with its own information, to its served SUs through a control channel such as the CPC using distribution methods such as in [8]. Subsequently, the SUs can access the spectrum. A summary of one round of the proposed algorithm is given in Table I.

The proposed algorithm can be implemented in a distributed manner, since, as already explained, the switch operation can be performed by the SBSs independently of any centralized entity. First, following the learning stage, the SBSs engage in pairwise negotiations, over the backbone, with their neighbors to identify possible switch operations. Given a present partition Π, for every SBS, the computational complexity of locating a switch operation, is $O(|\Pi|)$ in the worst case (the maximum value of $|\Pi|$ is $|\Pi| = N$ when the network is still non-cooperative). For performing a switch, each SBS needs to evaluate its potential utility in (6), which can be easily done as the SBS has knowledge of the metrics of its own channels and can obtain the rest of information needed to evaluate (6) from the SBSs of the candidate coalition by coordination over the backbone. Once a switch is identified, the SBS can leave its current coalition, and join the new coalition.

V. SIMULATION RESULTS AND ANALYSIS

For simulations, we set up a network composed of several cells, divided into various blocks composed of 3 adjacent cells. Each cell is a 1 km×1 km square with the SBS at the center.

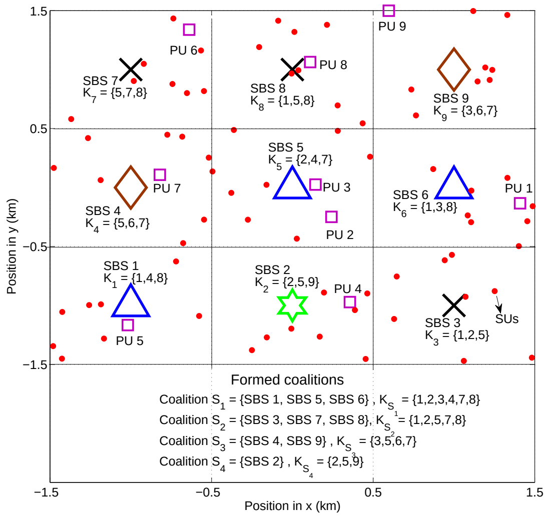

Fig. 1. A snapshot of a network resulting from the proposed algorithm with $N = 9$ SBSs (cells), $K = 9$ PUs, and 8 SUs per cell.

Within the area of a single cell, we randomly deploy 8 SUs per SBS and within the whole network, K PUs (transmitter-receiver pairs distant of at most 1 km) are deployed. Initially, we assume that each SBS $i \in \mathcal{N}$ can non-cooperatively learn and convey the statistics of $K_i = \frac{K}{3}$ channels (these non-cooperative K_i channels are randomly picked among the available PUs[1]). We set the transmit power of all the SUs to 10 mW, the transmit power of all the PUs to 100 mW, the path loss constant to $\mu = 3$, the target SNR to $\nu_0 = 10$ dB, the noise variance to $\sigma^2 = -90$ dBm, the threshold $\eta_i = \eta = 5, \forall i \in \mathcal{N}$ and the penalty factor to $\alpha_k = 50$, $\forall k \in \mathcal{K}$. The energy detection threshold $\lambda_{i,k}$ for an SBS i over channel k is chosen such that the false alarm probability $P^i_{f,k} = 0.05$, $\forall i \in \mathcal{N}$, $k \in \mathcal{K}$ which is a typical value in cognitive networks [2]. The time bandwidth product is set to a common value $m = 5$ [9].

In Fig. 1, we show a snapshot of the network structure resulting from the proposed algorithm for $N = 9$ SBSs/cells with $K = 9$ PUs randomly deployed. For figure clarity, the receivers of the PUs are not shown. Fig. 1 shows 4 different coalitions that formed among the various SBSs along with their known PUs. The partition in Fig. 1 is Nash-stable since no SBS has an incentive to switch its current coalition. For example, SBS 3 has a payoff of $\phi_3(S_2) = 13.17$ when being part of coalition $S_2 = \{3, 7, 8\}$, by switching to act non-cooperatively, this payoff drops to $\phi_3(\{3\}) = 8.89$. Similarly, if SBS 3 wants to switch from S_2 to join with SBS 2 or coalition $S_3 = \{4, 9\}$, its utility drops to $\phi_3(\{2, 3\}) = 8.95$ and $\phi_3(\{3, 4, 9\}) = 10.93$, respectively. Although by joining with coalition $S_1 = \{1, 5, 6\}$ SBS 3 improves its payoff from $\phi_3(S_2) = 13.17$ to $\phi_3(\{3, 1, 5, 6\}) = 15.54$, the members of S_1 do not agree on the joining of SBS 3 since, as a result, their payoffs would drop from $\phi_1(S_1) = 15.48$, $\phi_5(S_1) = 12.69$, $\phi_6(S_1) = 19.1$ to $\phi_1(\{3, 1, 5, 6\}) = -69.38$, $\phi_5(\{3, 1, 5, 6\}) = 9.31$, $\phi_6(\{3, 1, 5, 6\}) = 18.83$. This drop is a consequence of low trust values between SBS 3 and the members of S_1, notably SBS 1.

In Fig. 2, we show the average payoff achieved per SBS

[1]This method of selection is considered as a general case, other methods for non-cooperatively picking the PUs can also be accommodated.

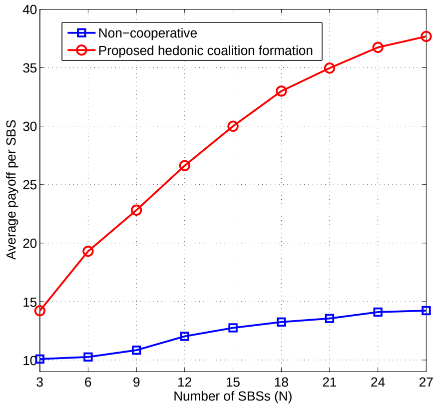

Fig. 2. Average payoff per SBS vs. number of SBSs (cells) N for a network with $K = 15$ PUs.

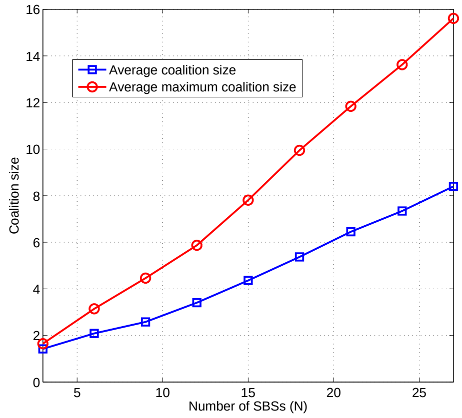

Fig. 3. Average and average maximum coalition size vs. number of SBSs (cells) N for a network with $K = 15$ PUs.

for a network with $K = 15$ channels as the number of SBSs in the network, N, increases. The results are averaged over random positions of all nodes (SUs and PUs) and the random realizations of the probabilities $\theta_k, \forall k \in \mathcal{K}$. Fig. 2 shows that, as the number of SBSs N increases, the performance of both the proposed scheme and the non-cooperative scheme increases. However, the slope of increase of the performance of the non-cooperative scheme is slower than that of the proposed algorithm. For the non-cooperative case, this increase is a *sole* result of the fact that, as N increases, a more diverse selection of channels is possible non-cooperatively. For the proposed algorithm, the increase in the average payoff per SBS with N is due to the increased possibility of finding better cooperating partners and sharing more channels among the SBSs. At all network sizes, the proposed hedonic coalition formation algorithm maintains a significant performance advantage compared to the non-cooperative case, increasing with N, and reaching up to 165% relative to the non-cooperative case at $N = 27$ SBSs.

In Fig. 3, we show the average and average maximum coalition size (averaged over the random positions of the PUs and SUs and the random realizations of the probabilities $\theta_k, \forall k \in \mathcal{K}$) resulting from the proposed hedonic coalition formation algorithm as the number of SBSs, N, increases, for a network with $K = 15$ channels. Fig. 3 shows that both the average and average maximum coalition size increase with the number of SBSs. This is mainly due to the fact that as N increases, the number of candidate cooperating partners increases. Through Fig. 3 we note that the formed coalitions have a moderate to large size, with the average and maximum coalition size, respectively, ranging from around 1.4 and 1.6 at $N = 3$ to around 8.4 and 15.6 at $N = 27$. Using hedonic coalition formation among SBSs, the resulting network is mainly composed of a small number of relatively large coalitions rather than a large number of small coalitions.

VI. Conclusions

In this paper, we proposed a novel cooperative scheme allowing the SBSs in a cognitive radio network to share their knowledge of the spectrum, and, hence convey better information (for the SUs) over their control channels such as the cognitive pilot channel (CPC). We modeled the problem as a hedonic coalition formation game and we derived an algorithm for coalition formation. Using the proposed algorithm, each SBS can take an individual decision to join or leave a coalition, while maximizing its utility. The utility of every SBS accounts for the benefits from learning new channels and the costs from receiving inaccurate information. Further, we showed that, using hedonic coalition formation, the SBSs can self-organize into a Nash-stable partition. Simulation results show that the proposed algorithm improves the average payoff per SBS up to 165% relative to the non-cooperative case for a large network of 27 SBSs. Future work can consider, among others, joint collaborative spectrum sensing, advanced SUs multiple access schemes and the impact of the delay for sending the CPC data.

References

[1] S. Haykin, “Cognitive radio: brain-empowered wireless communications,” *IEEE J. Select. Areas Commun.*, vol. 23, pp. 201–220, Feb. 2005.

[2] D. Niyato, E. Hossein, and Z. Han, *Dynamic spectrum access and management in cognitive radio networks*. Cambridge, UK: Cambridge University Press, 2009.

[3] S. Huang, X. Liu, and Z. Ding, “Optimal sensing-transmission structure for dynamic spectrum access,” in *Proc. Int. Conf. on Computer Communications (INFOCOM)*, Rio de Janeiro, Brazil, Apr. 2009.

[4] D. Raychaudhuri and X. Jing, “A spectrum etiquette protocol for efficient coordination of radio devices in unlicensed bands,” in *Proc. of IEEE Int. Symp. on Personal, Indoor and Mobile Radio Communications (PIMRC)*, Beijing, China, Sep. 2003.

[5] P. Houz, S. B. Jemaa, and P. Cordier, “Common pilot channel for network selection,” in *Proc. of IEEE Vehicular Technology - Spring*, Melbourne, Australia, May 2006.

[6] M. Filo, A. Hossain, A. R. Biswas, and R. Piesiewicz, “Cognitive pilot channel: Enabler for radio systems coexistence,” in *Proc. of Second Int. Workshop on Cognitive Radio and Advanced Spectrum Management*, Aalborg, Denmark, May 2009.

[7] O. Sallent, J. Perez-Romero, R. Agusti, and P. Cordier, “Cognitive pilot channel enabling spectrum awareness,” in *Proc. Int. Conf. on Communications, Workshop on*, Dresden, Germany, Jun. 2009.

[8] J. Perez-Romero, O. Salient, R. Agusti, and L. Giupponi, “A novel on-demand cognitive pilot channel enabling dynamic spectrum allocation,” in *Proc. of IEEE Int. Symp. on New Frontiers in Dynamic Spectrum Access Networks (DySPAN)*, Dublin, Ireland, Apr. 2007.

[9] A. Ghasemi and E. S. Sousa, “Collaborative spectrum sensing for opportunistic access in fading environments,” in *IEEE Symp. New Frontiers in Dynamic Spectrum Access Networks*, Baltimore, USA, Nov. 2005.

[10] J. Proakis, *Digital Communications*. McGraw-Hill, 4th edition, 2000.

[11] Y. Sun, W. Yu, Z. Han, and K. J. R. Liu, “Information theoretic framework of trust modelling and evaluation for ad hoc networks,” *IEEE Journal on Selected Areas in Communications*, vol. 24, no. 2, pp. 305–317, Feb. 2006.

[12] R. B. Myerson, *Game Theory, Analysis of Conflict*. Cambridge, MA, USA: Harvard University Press, 1991.

[13] W. Saad, Z. Han, M. Debbah, A. Hjørungnes, and T. Başar, “Coalition game theory for communication networks: A tutorial,” *IEEE Signal Processing Mag., Special issue on Game Theory in Signal Processing and Communications*, vol. 26, no. 5, pp. 77–97, Sep. 2009.

[14] D. Ray, *A Game-Theoretic Perspective on Coalition Formation*. New York, USA: Oxford University Press, 2007.

[15] A. Bogomonlaia and M. Jackson, “The stability of hedonic coalition structures,” *Games and Economic Behavior*, vol. 38, pp. 201–230, Jan. 2002.

[16] C. J. L. Arachchige, S. Venkatesan, and N. Mittal, “An asynchronous neighbor discovery algorithm for cognitive radio networks,” in *Proc. IEEE Symp. on New Frontiers in Dynamic Spectrum Access Networks (DySPAN)*, Chicago, IL, USA, Oct. 2008.