# **Defense Presentation**





# Hybrid Quantum-Classical Optimization for Data Center Energy System

Ph.D. Candidate: Zhongqi Zhao

Date: 07/18/2025

Academic Advisor: Dr. Lei Fan and Dr. Zhu Han

Committee Member:

Dr. Miao Pan, Dr. David Mayerich, and Dr. Xiaodi Wu

### **Outline**



### Introduction

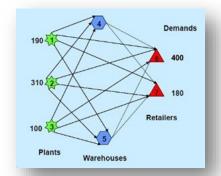
- Mixed-integer Linear Programming & Quantum Computing
- Work 1: Hybrid Quantum Benders' Decomposition (HQC-Bend) for Mixed-integer Linear Programming and Python Package
- Work 2: Energy Management Problem in Internet Data Center Using HQC-Bend
- Work 3: Optimal Energy and LLM Training Job Scheduling for Internet Data Center Using Nonlinear HQC-Bend.
- Future Work & Conclusion

# Introduction (1/7) – MILP Application

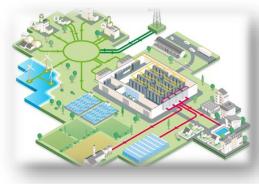




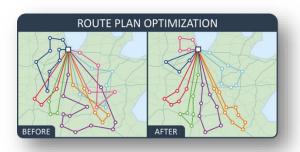
☐ (Mixed)-integer Linear Programming



Production & Demand.



**Energy Management** 

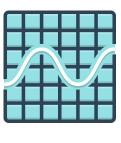


**Route Planning** 





State

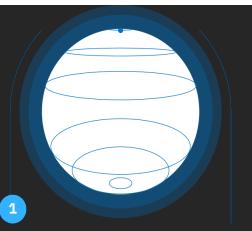


**Amplitude** 

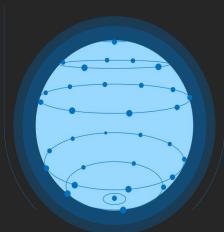
# Introduction (2/7) – Quantum Computing (QC)



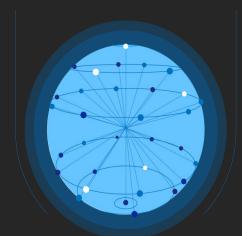


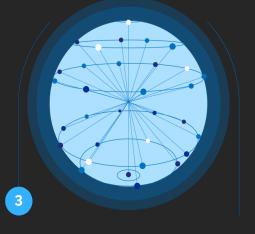


Activate the spread



**Encode the problem** 





Unleash the power



### Quantum computers CAN

create vast multidimensional spaces in very LARGE problems.

translate them back into what we CAN use And understand

# Classical computers

to achieve the same.

# Introduction (3/7) - Quantum Computing (QC)

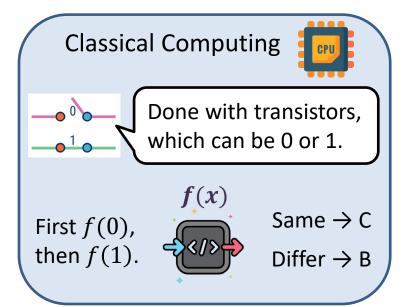




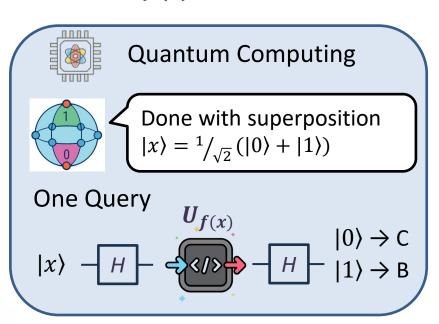
☐ What is Quantum Computing (QC):

harnesses the principles of quantum mechanics—superposition, entanglement, and interference—to process information in parallel and probabilistic ways, enabling solutions to complex problems.

• A Toy Example: "Mystery Coin" is Heads or Tails f(x)





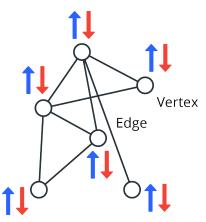


# Introduction (4/7) – QC vs. Classical Computing



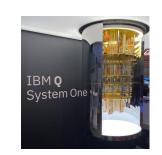


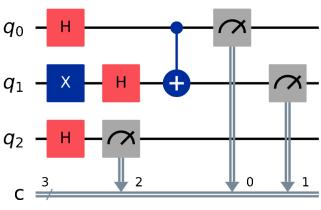
?	Advantage Advantage	<b>Disadvantage</b>
CPU (Central Processing Unit)	Versatility to Classical Algorithms	<ul> <li>Limited Parallelism -&gt; Slower*</li> </ul>
QPU (Quantum Processing Unit)	Quantum Parallelism/Tunneling	Limited & Specialized Cases



← Quantum Annealing (QA)
(QDC) Digital Quantum Circuit ⇒





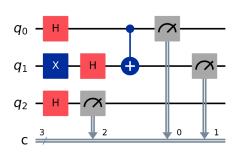


# Introduction (5/7) – QA vs. DQC (MFG)





### **Digital Quantum Circuit**





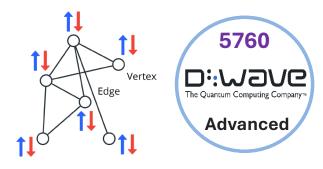
Continuous/Discrete (VQA, QAOA)\*



: Better performance is not Guaranteed.



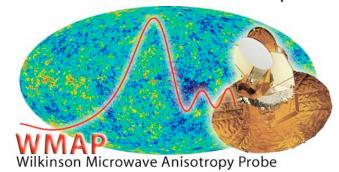
### **Quantum Annealing**



Discrete (QUBO, Ising)



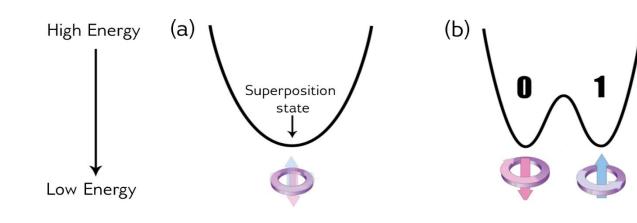
: Need to translate/reformulate problem

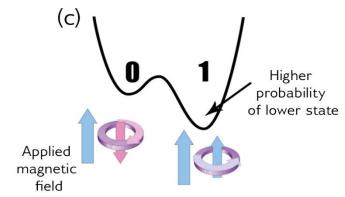


# Introduction (6/7) – QA vs. DQC (MFG)









A matter of microseconds.



- Initial Qubits
- Superposition at  $|0\rangle$ s and  $|1\rangle$ s.
- · Not yet coupled.

- Qubits are entangled.
- At state of many possible answers.
- Couplers & biases applied

- Inputs' energy are set.
- Lowest energy is at or closes to the optima.
- Energy → possibility

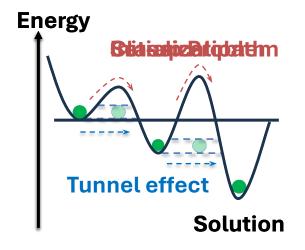
# Introduction (7/7) – QA in D-WAVE





Core idea: Encoding the objective function as the eigenvalue of the final ground state of Schrodinger equation, based on Adiabatic Quantum Computing Model

 $\sigma \in \{-1,1\}$ 



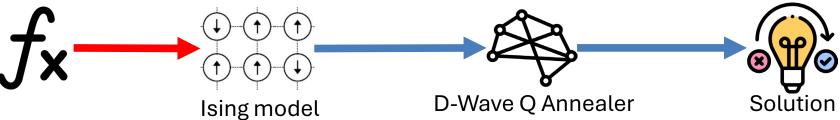
$$H_{ ext{ising}} = - \underbrace{rac{A(s)}{2} \Biggl( \sum_i \hat{\sigma}_x^{(i)} \Biggr)}_{ ext{Initial Hamiltonian}}$$

$$+ \; rac{B(s)}{2} \Biggl( \sum_i h_i \hat{\sigma}_z^{(i)} + \sum_{i>j} J_{i,j} \hat{\sigma}_z^{(i)} \hat{\sigma}_z^{(j)} \Biggr) \; .$$

 $Final\ Hamiltonian$ 

Spins interact with applied field

Neighboring spins interact with each other



### **Outline**



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- Future Work & Conclusion

# Work I: HQC-Bend for MILP & Software





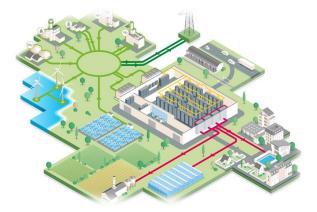
### ☐ Motivation: Internet Data Center (IDC) Energy Management is Vital!

Renewable energy resources is many



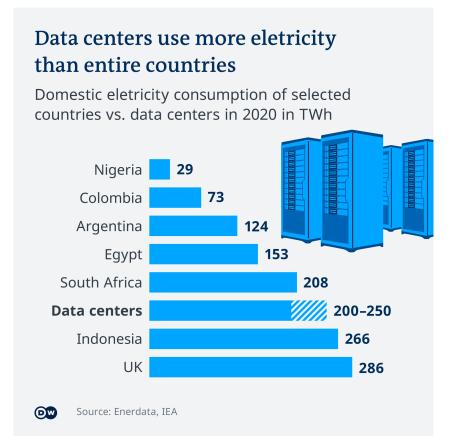






IDC server room devices are complex

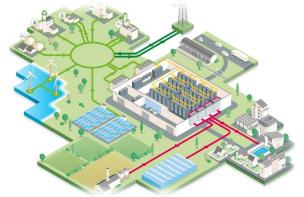


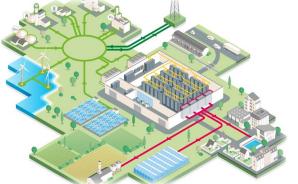






■ Motivation: Internet Data Center (IDC) Energy Management is Vital!







### Binary variables: $x \in \mathbb{B}$















### Continuous variables: $y \in \mathbb{R}$













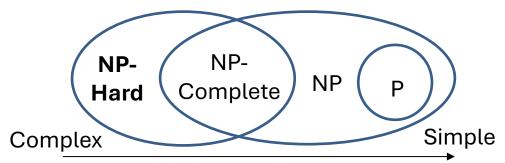




☐ (Mixed)-integer linear programming

$$egin{array}{lll} \max_{\mathbf{x},\mathbf{y}} & \mathbf{c}^{\mathsf{T}}\mathbf{x} + \mathbf{h}^{\mathsf{T}}\mathbf{y} & \max_{\mathbf{x},\mathbf{y}} & \mathbf{c}^{\mathsf{T}}\mathbf{x} \\ \mathrm{s.t.} & \mathbf{T}\mathbf{x} \leqq \mathbf{p} & \mathrm{s.t.} & \mathbf{A}\mathbf{x} \leqq \mathbf{b} \\ & \mathbf{A}\mathbf{x} + \mathbf{G}\mathbf{y} \leqq \mathbf{b} & \mathbf{x} \in \mathbb{Z}^n. \end{array}$$

- MILP is NP-Hard,
- It can't be solved in polynomial time\*.



Problem type	Example Problem			
NP-Hard	Turing Halting Problem (M)ILP			
NP-Complete	Graph 3-coloring			
NP	Factoring			
Р	Graph Connectivity			

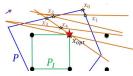
Enumerative Methods



Branch and Bounds



Cutting Planes Methods







# Consider a Mixed-Binary LP on right-hand-side, Classical Benders' Decomposition Algorithm is:

- 1) Solve the master problem (MAP). Obtain solution  $\overline{x}$  and  $\overline{\lambda}$ .
- 2) Determine  $\underline{\lambda}$  by solving the dual of the subproblem (**DSUB**).
- 3) If DSUB is unbounded. Add the corresponding feasibility cuts to MAP and return to Step 1. (**Feasibility Cuts**).
- 4) If the DSUB objective value  $< \lambda$  and finite, Add the Optimality Cuts to MAP and return to Step 1. (Optimality Cuts)
- 5) If  $f(|\bar{\lambda} \underline{\lambda}|) \le \tau$ . then we recognize the current  $\bar{x}$  solves the original mixed integer program, with optimal y equal to the solution to the primal subproblem with  $x = \bar{x}$ .

# $egin{array}{l} \max_{\mathbf{x},\mathbf{y}} \ \mathbf{c}^{\intercal}\mathbf{x} + \mathbf{h}^{\intercal}\mathbf{y} \ & ext{s.t.} \ \mathbf{T}\mathbf{x} \leqq \mathbf{p} \ & \mathbf{A}\mathbf{x} + \mathbf{G}\mathbf{y} \leqq \mathbf{b} \ & \mathbf{x} \in \mathbb{Z}^n, \mathbf{y} \in \mathbb{R}^m \end{array}$

$$\max_{\mathbf{x},\lambda} \mathbf{c}^\intercal \mathbf{x} + \lambda \ \mathrm{s.t.} \ \mathbf{Tx} \leqq \mathbf{p}$$
 Master Problem

$$egin{aligned} (\mathbf{b} - \mathbf{A} \mathbf{x})^\intercal u^k &\geq \lambda & ext{ for } k \in K \ (\mathbf{b} - \mathbf{A} \mathbf{x})^\intercal r^j &\geq 0 & ext{ for } j \in J \ \lambda \in \mathbb{R}, \mathbf{x} \in \mathbb{B}^n \end{aligned}$$

Feasibility Cuts or Optimality Cuts

Binary solution **x** 

$$egin{array}{ll} \min_{\mathbf{u}} & (\mathbf{b} - \mathbf{A}\mathbf{x})^{\mathsf{T}}\mathbf{u} \\ \mathrm{s.t.} & \mathbf{G}^{\mathsf{T}}\mathbf{u} \geq \mathbf{h} & \mathsf{DSUB} \\ & \mathbf{u} \in \mathbb{R}^m_{\scriptscriptstyle \perp} \end{array}$$





**Algorithm 1** Hybrid Quantum-Classical Benders' Decomposition Algorithm

```
Require: Initial sets \hat{K} of extreme points and \hat{J} of extreme
   rays of Q
   \overline{\lambda} \leftarrow +\infty
   \lambda \leftarrow -\infty
   while |\overline{\lambda} - \underline{\lambda}| \ge \epsilon do
         P \leftarrow Appropriate penalties numbers or arrays
         Q \leftarrow Reformulate both objective and constraints and
   construct the QUBO formulation by using corresponding
   rules
         x' \leftarrow Solve MAP by quantum computer.
         \overline{\lambda} \leftarrow \text{Extract } \mathbf{w} \text{ and replace the } \overline{\lambda} \text{ with } \overline{\lambda} (\mathbf{w})
         z_{LP}(\mathbf{x}) \leftarrow \text{Solve the DSUB problem}
         \underline{\lambda} \leftarrow z_{LP}(\mathbf{x})
         if z_{LP}(\mathbf{x}) = -\infty then
               An extreme ray j of Q is found (Feasibility Cut).
               \hat{J} = \hat{J} \cup \{j\}
         else if z_{LP}(\mathbf{x}) < \overline{\lambda} and \overline{\lambda} \neq +\infty then
               An extreme point k of Q is found (Optimality Cut).
              \hat{K} = \hat{K} \cup \{k\}
         end if
   end while
```

$$f(\mathbf{x}') = \mathbf{x}'^\mathsf{T} \mathbf{Q}_{\mathrm{QUBO}} \mathbf{x}'$$
 Obj.  $\mathbf{x}^\mathsf{T} \operatorname{diag}(\mathbf{c}) \mathbf{x}$  Connection  $+ \sum_{i=-m}^{\overline{m}_+} w_{i+\underline{m}} 2^i w_{i+\underline{m}} - \sum_{j=0}^{\overline{m}_-} w_{j+(1+\underline{m}+\overline{m}_+)} 2^j w_{j+(1+\underline{m}+\overline{m}_+)}$  Var.  $+ \sum_{k \in K} P_k \left( \overline{\lambda}(\mathbf{w}) + (u^k)^\mathsf{T} \mathbf{A} \mathbf{x} + \sum_{l=0}^{\overline{l}^K} 2^l s_{kl}^K - b^\mathsf{T} u^k \right)^2$  Feas. Cut  $+ \sum_{j \in J} P_j \left( (r^j)^\mathsf{T} \mathbf{A} \mathbf{x} + \sum_{l=0}^{\overline{l}^J} 2^l s_{kl}^J - \mathbf{b}^\mathsf{T} r^j \right)^2$ 

Integer solution

NP-hard
Integer Variables
Master Problem

Subproblem
Continuous Variables
Polynomial Complexity

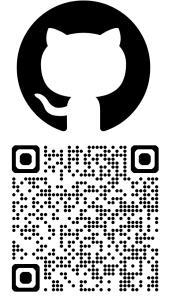
Feasibility Cuts & Optimality Cuts

return  $\lambda$ , x

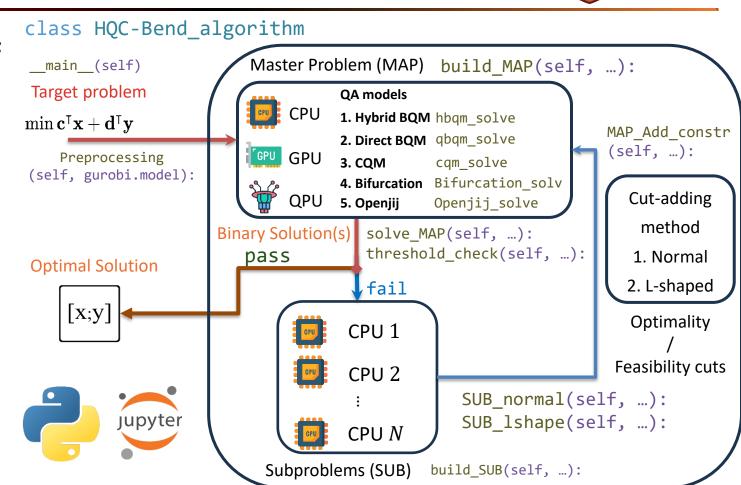








GitHub Page <a href="https://github.com/dj">https://github.com/dj</a> zts/HQCMCBD-API







```
(Target Model) \min \mathbf{c}^{\mathsf{T}} \mathbf{x} + \mathbf{d}^{\mathsf{T}} \mathbf{y}
```

- s.t.  $\mathbf{A}\mathbf{x} + \mathbf{G}\mathbf{y} \leq \mathbf{b}$ ,  $\mathbf{H}\mathbf{x} \leq \mathbf{e}$ ,  $\mathbf{x} \in \mathbb{B}^n, \mathbf{y} \in \mathbb{R}^m$ .
- Preprocessing
  (self, gurobi.model):

3

 $\mathbf{c}, \mathbf{H}, \mathbf{e}$ 

3

d, A, G, b

# $\begin{array}{c} \mathsf{build\_MAP}(\mathsf{self}) \colon \\ (\mathsf{MAP}) \; \min_{\mathbf{x}, \lambda, \mathbf{z}} \mathbf{c}^\intercal \mathbf{x} + \lambda \end{array}$

s.t.  $\mathbf{H}\mathbf{x} \leq \mathbf{e}$ ,

 $f(\lambda,\mathbf{x}) \leq 0, ext{ (Optimality cuts)},$ 

 $f(\mathbf{x}) \leq 0$ , (Feasibility cuts),

 $\lambda = g(\mathbf{z}), (\text{Discretization}),$ 

 $\mathbf{x} \in \mathbb{B}^n, \lambda \in \mathbb{Q}, \mathbf{z} \in \mathbb{B}^l.$ 

### (SUBs)

build\_SUB
(self, count):

Preprocessing of the Target Model

the target optimization model is decomposed into multiple subproblems.

- **1** Decompose Target Model:
- 2 Preprocessing:
- Using *Gurobi* to extract **key** components:
- **3** Problems Construction:
- MAP (Master Problem):
- SUB (Subproblems):







CPU: Classical Gurobi solver (1-cut), Simulated Quantum Annealing Openjij Solver (1-cut)

**GPU**: Simulated Bifurcation (1-cut)









Simulated Bifurcation

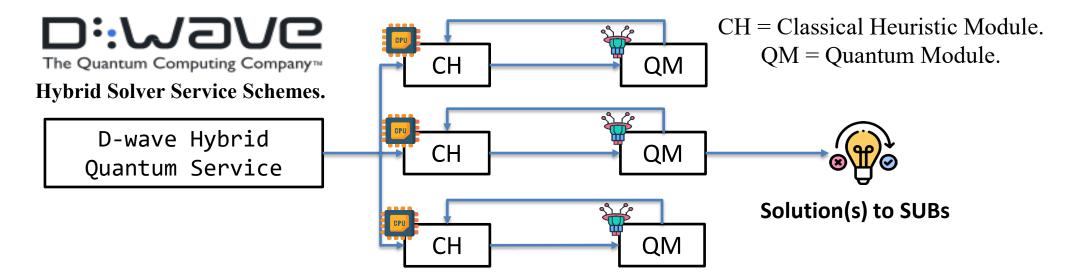
**Quantum Annealing Models** 

D-wave Quantum Service

D-wave Hybrid Quantum Service



1. **Hybrid BQM**: hbqm\_solve(self) 2. *Direct BQM*: qbqm\_solve(self) 3. **CQM**: cqm\_solve(self)









# Subproblem Solving Logic Flow

SUB normal:

#### **Normal**

SUB\_lshape:

L-shape

Optimality Cut (OC)

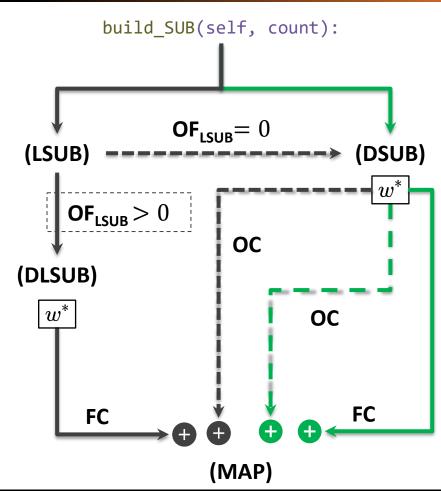
$$(\mathbf{b} - \mathbf{A}\mathbf{x})^{\mathsf{T}}\mathbf{w}^* \leq \lambda$$

Feasibility Cut (FC)

$$(\mathbf{b} - \mathbf{A}\mathbf{x})^{\intercal}\mathbf{w}^* \leq 0$$

Objective Function of \* (OF\*)

$$f_{ ext{obj},*}(\mathbf{x},\mathbf{y})$$





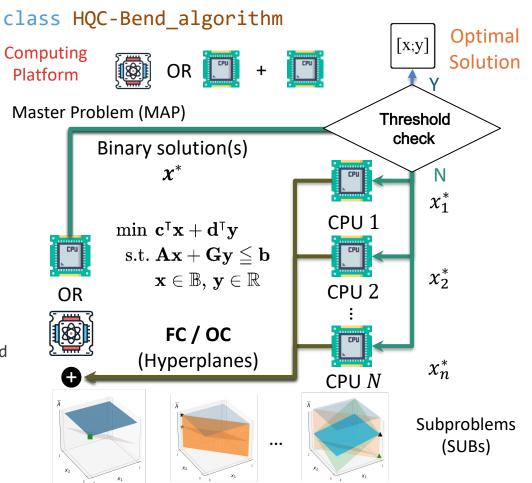


### **Example Python Code**

```
import gurobipy as gp
from gurobipy import GRB, Model, quicksum
import numpy as np
import sys
%run HQC-Bend_notebook.ipynb
# Create a new Gurobi model
model = gp.Model("Example")
# Set the objective function
model. setObjective (c@x+h@y,GRB.MAXIMIZE)
# Add the constraints
model.addConstr(A@x+G@y<=b,name="constraints")
# Optimize the model
model.optimize()
# call the solver
Solver = HQC-Bend algorithm(model, mode = "default")
Solver.run()
```

#### **Example Text Output**

The n-th Config file of quantum sampling is created successfully at F:\...\Dwave info-round-n.json. create optimality cut 2.create optimality cut Round n: Current Obj. value is 9.0; lambda\_upper is 17.0; lambda\_lower is 11.0; Relative gap is 54.545%; Absolute gap is 6.0.



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$$\min_{\substack{u_t^{ ext{dis}},\ u_t^{ ext{chr}},p_t^{ ext{dis}},\ p_t^{ ext{chr}},x_{j,t}^{ ext{chiller}},\ x_t^{ ext{cower}},T_{i,t}^{ ext{Zone}},T_{t}^{ ext{sup}},v_t^{ ext{vent}}},\sum_{t=0}^T p_t^{ ext{e,g}} e_t^{ ext{dc,in}}. \qquad x \in \mathbb{B}, y \in \mathbb{R}$$

The objective function: minimize the total cost of electricity imported from the grid.

$$e_t^{ ext{dc,in}} = E_t^{ ext{HVAC}} + E_t^{ ext{Server}} + \Delta E_t^{ ext{B}} - E_t^{ ext{Solar}}, \; orall t.$$

The sum of every energy sources and consumers.

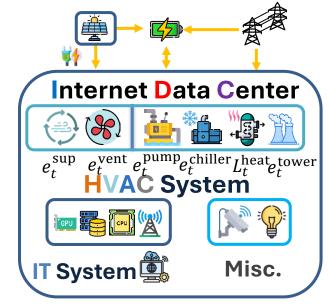
$$E_t^{ ext{HVAC}} = e_t^{ ext{sup}} + e_t^{ ext{vent}} + e_t^{ ext{chiller}} + e_t^{ ext{pump}} + e_t^{ ext{tower}}, \ orall t.$$

The sum of every parts' energy consumption.

$$\Delta E_t^{
m B} = p_t^{
m chr} \eta^{
m chr} - p_t^{
m dis} \cdot (\eta^{
m dis})^{-1}, \ orall t.$$



Battery's (dis)charging law



$$\underline{\xi^{\mathrm{B}}} \leq E_{t+1}^{\mathrm{B,state}} \leq \overline{\xi^{\mathrm{B}}}, \ orall t$$

Battery status requirements at time t

$$E_{t+1}^{ ext{B,state}} = E_{t}^{ ext{B,state}} + \Delta E_{t}^{ ext{B}}, \; orall t.$$

Battery status at time t.





#### **Detailed Formulation**

$$T_{i,t}^{\mathrm{Zone},-} \leq T_{i,t}^{\mathrm{Zone}} \leq T_{i,t}^{\mathrm{Zone},+}, \ orall i,t.$$

Upper/lower bound of room temperature

$$v_t^{ ext{vent}} + v_t^{ ext{out}} \ge \underline{v}_t^{ ext{vent}}, \ \forall t.$$



The minimum ventilation air flow speed

$$v_t^{ ext{sup}} = v_t^{ ext{out}} + v_t^{ ext{return}}, \ orall t.$$

The air flow speed that comes out of the AC

$$\sum_{j \in \mathbf{T}^{ ext{chiller}}} x_{j,t}^{ ext{chiller}} m_{j,t}^{ ext{chiller}} \, \geq m_t^{ ext{chw}} \, , \; orall t.$$



$$\sum_{j \in \mathbf{I}^{ ext{tower}}} x_{j,t}^{ ext{tower}} m_{j,t}^{ ext{tower}} \geq m_t^{ ext{conw}} \,, \; orall t.$$



min capacity of chiller/condense tower water

$$T_{i,t}^{\sup,-} \leq T_{i,t}^{\sup} \leq T_{i,t}^{\sup,+}, \ \forall i,t.$$

Upper/lower bound of AC temperature

$$egin{aligned} L_t^{ ext{heat}} &= \left(T_t^{ ext{out}} - \sum_{i \in \mathbf{I}^{ ext{Zone}}} \chi_i oldsymbol{T}_{i,t}^{ ext{sup}} 
ight) \cdot v_t^{ ext{out}} \, c_p^{ ext{air}} \ &+ \sum_{i \in \mathbf{I}^{ ext{Zone}}} \chi_i \Big(oldsymbol{T}_{i,t}^{ ext{Zone}} - oldsymbol{T}_{i,t}^{ ext{sup}} \Big) \cdot v_t^{ ext{return}} c_p^{ ext{air}} \;, \; orall t. \end{aligned}$$

The sum of heat load in data center

$$egin{aligned} m_t^{ ext{chw}} &= rac{L_t^{ ext{heat}}}{\left(T_t^{ ext{chwr}} - T_t^{ ext{chws}}
ight) \cdot c_p^{ ext{water}}}, \, orall t. \ m_t^{ ext{conw}} &= rac{L_t^{ ext{heat}}}{\left(T_t^{ ext{conwr}} - T_t^{ ext{conws}}
ight) \cdot c_n^{ ext{water}}}, \, orall t. \end{aligned}$$

The amount of chiller/condense tower water to take away the heat.





#### **Detailed Formulation**

$$egin{aligned} e_t^{ ext{chiller}} &= \sum_{j \in \mathbf{I}^{ ext{chiller}}} x_{j,t}^{ ext{chiller}} ig(eta_{0,j}^{ ext{chiller}} + eta_{1,j}^{ ext{chiller}} m_{j,t}^{ ext{chiller}} ig), \ orall t. \ e_t^{ ext{tower}} &= \sum_{j \in \mathbf{I}^{ ext{tower}}} x_{j,t}^{ ext{tower}} ig(eta_{0,j}^{ ext{tower}} + eta_{1,j}^{ ext{tower}} m_{j,t}^{ ext{tower}} ig), \ orall t. \end{aligned}$$

Energy consumption of chillers & condense towers

$$e_t^{ ext{pump}} = eta_0^{ ext{pump}} + eta_1^{ ext{pump}} m_t^{ ext{conw}}, \ orall t.$$



Energy consumption of pump in condense towers

$$egin{aligned} & \max_{\mathbf{x}, \mathbf{y}} \ \mathbf{c}^{\intercal}\mathbf{x} + \mathbf{h}^{\intercal}\mathbf{y} \ & ext{s.t.} \ \mathbf{T}\mathbf{x} \leqq \mathbf{p} \ & \mathbf{A}\mathbf{x} + \mathbf{G}\mathbf{y} \leqq \mathbf{b} \ & \mathbf{x} \in \mathbb{Z}^n, \mathbf{y} \in \mathbb{R}^m \end{aligned}$$

 $v_t^{ ext{sup}} = v_t^{ ext{out}} + v_t^{ ext{return}}, \ orall t.$ 



$$e_t^{ ext{vent}} = eta_0^{ ext{vent}} ig( v_t^{ ext{vent}} - \underline{v}^{ ext{vent}} ig), \ orall t.$$

Energy consumption for ventilation

$$v_t^{ ext{vent}} \geq \underline{v}^{ ext{vent}}, \ \forall t.$$

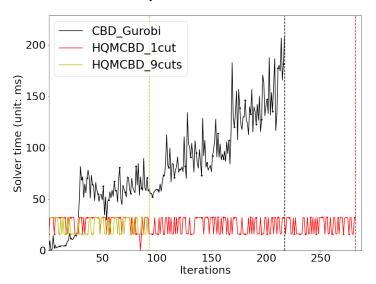
Ventilation Requirement

Mixed-integer linear programming (MILP)





Simulation: Comparison Between Classical solver, HQC-Bend with different multi-cuts options



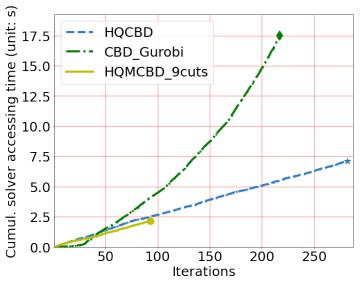


Table 1. Iteration Comparison Between **HQC-Bend** with different multi-cuts strategies

	Set-up	x	Iter. of CBD	Aver. iter. of HQCMBD $(N=3)$	Gain		Iter. of $QCMB$ (N = 6)	D	Aver. iter. of HQCMBD $(N=6)$	Gain	Aver. iter. of HQCMBD $(N=9)$	Gain
Case 1	${3,4,5}$	33	117	83.67	-28%	66	74	65	68.33	-42%	56	-52%
Case 2	$\{4, 2, 2\}$	24	217	160	-26%	120	125	127	127.33	-41%	100	-54%

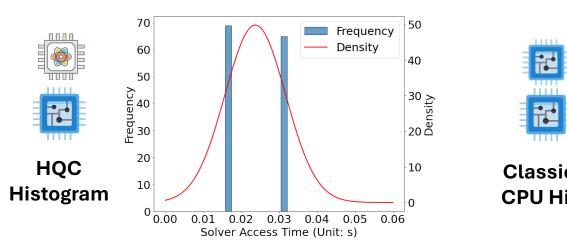


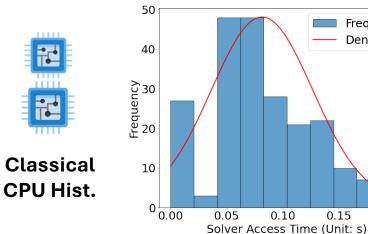
Frequency

0.20

Density







**Table 2. Standard Deviation Comparison** 

Detail Model	Standard Deviation Unit: 10 <sup>-3</sup>	Gain
Case1 CBD	CBD 186.0	
Casel HQCMBD	6.8	96.33%

Detail Model	Standard Deviation Unit: 10 <sup>-3</sup>	Gain
Case2 CBD	45.2	82.31%
Case2 HQCMBD	8.0	02.0170

The HQC-Bend **outperforms** the classical approach in terms of

solver access time, Iterations, and robustness.

### **Outline**



- Introduction
- Work 1: Hybrid Quantum Benders' Decomposition (HQC-Bend) for Mixed-integer
   Linear Programming and Python Package
- ◆ Work 2: Energy Management Problem in Internet Data Center Using HQC-Bend
- Work 3: Optimal Energy Management and LLM Training Job Scheduling for Internet
   Data Centers Using Nonlinear HQC-Bend.
- Future Work & Conclusion







Motivation: LLM training is a core part of IDC.

1300MWh











Is the previous work good enough?

US homes (130) Annually

Work 3

Improvement?

**IDC Number** 



Single

 $\geq 2$ , with data link



Power consumption



Constant

Variable to LLM tasks



**LLM Training** Task Scheduling















Background: Token, Reward and Computation Resource.

Dr. Zhu Han is widely recognized as a pioneering force in the fields of wireless communications, game theory, quantum computing and network science. (GPT-40)

<|im\_start|>system<|im\_sep|>You are a helpful assistant<
|im\_end|><|im\_start|>user<|im\_sep|>Dr. Zhu Han is widely
recognized as a pioneering force in the fields of wirele
ss communications, game theory, quantum computing and ne
twork science. H<|im\_end|><|im\_start|>assistant<|im\_sep|
>

200264, 17360, 200266, 3575, 553, 261, 10297, 29186, 200 265, 200264, 1428, 200266, 5822, 13, 151904, 21513, 382, 20360, 20418, 472, 261, 107046, 9578, 306, 290, 8532, 32 8, 25556, 24296, 11, 2813, 17346, 11, 48889, 34349, 326, 5402, 11222, 13, 487, 200265, 200264, 173781, 200266

$$T_{j,n}^{job} \approx \frac{6 \times N \times d_{\text{model},j}}{\text{n FLOPS}}$$



Performance Evaluation (GPT-like model)

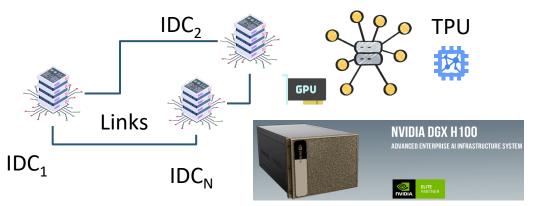
 $ext{FLOPs per token} pprox 6 imes N imes d_{ ext{model}}^2$ 

N is the number of transformer layers  $d_{
m model}$  is the hidden dimension (model width)

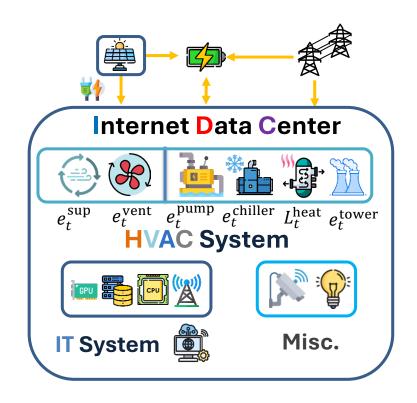




System Model: Multi-IDC LLM Task Scheduling and Energy Management



Param	LLM T	ask	s Pool	Electricity Price
IDC <sub>1</sub>	LLM <sub>11</sub>		LLM <sub>1M</sub>	$oldsymbol{c}^e_{1,T}$
IDC <sub>2</sub>	LLM <sub>21</sub>		LLM <sub>2M</sub>	$\boldsymbol{c}^{e}_{2,T}$
$IDC_N$	LLM <sub>N1</sub>		$LLM_{NM}$	$oldsymbol{c}_{N,T}^e$



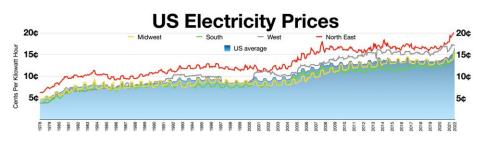




System Model: Multi-IDC LLM Task Scheduling and Energy Management

### □ Challenges:

- LLM-Task-model-scheduling-wise:
- Maximize the net income;
- Local industrial electricity Price;
- Time-sensitive LLM task completion;
- Limited task data transmission link, which to use?
- Computing nodes with different performance,
   which to use?
- & concerns in Work 3 in multiple IDC locations.







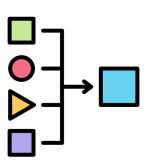




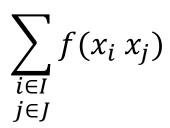


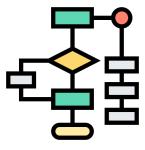
- ☐ Challenges:
  - Algorithm-wise
  - Concerns 1: Generality of the model
  - Concerns 2: Parameters selection
  - Concerns 3: Nonlinearity in Obj. Function
  - Concerns 4: Creating the algorithm for

Mixed-integer nonlinear programming.













Problem Formulation: Maximize the net profit of IDCs over a period. (Binary, Continuous)

$$egin{aligned} \max \ c^{ ext{profit}} - c^{ ext{loss}} - c^{ ext{transfer}} - c^{ ext{ebill}}, \ c^{ ext{profit}} &= \sum_{j \in \mathcal{J}} C_j^{ ext{profit}} x_j^{ ext{Done}}, \ c^{ ext{loss}} &= \sum_{j \in \mathcal{J}} C_j^{ ext{loss}} x_j^{ ext{Abort}}, \ c^{ ext{transfer}} &= \sum_{j \in \mathcal{J}} \sum_{n \in \mathcal{N}} C_{j,n}^{ ext{transfer}} & x_j^{ ext{TF}} \cdot x_{j,n}^{ ext{job}}. \ c^{ ext{ebill}} &= \sum_{i \in \mathcal{I}} \sum_{t \in \mathcal{T}} C_{i,t}^{ ext{ebill}} e_{i,t}^{ ext{G}}, \ x_j^{ ext{Done}} + x_j^{ ext{Abort}} + x_j^{ ext{Hold}} &= 1, \ orall j \in \mathcal{J}, \ x_j^{ ext{Hold}} &= 0, & ext{if } j \in \mathcal{J}^{ ext{NTS}}. \ x_j^{ ext{Abort}} &= 0, & ext{if } j \in \mathcal{J}^{ ext{NTS}}. \end{aligned}$$

Objective function

**Profit decomposition** 

Monetary loss decomposition

Job transfer cost decomposition (nonlinearity)

Electricity bill for IDC operation







### 1. Job scheduling (Universal)

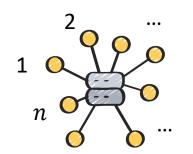
$$\sum
olimits_{n \in \mathcal{N}} x_{j,n}^{ ext{job}} = x_j^{ ext{Done}}, \ orall j \in \mathcal{J},$$

Search all nodes to see whether the task is finished

$$u_{j,n,t}^{ ext{job}} \leq x_{j,n}^{ ext{job}}, \ orall j \in \mathcal{J}, t \in \mathcal{T}, n \in \mathcal{N},$$

$$egin{aligned} u_{i,n,t-1}^{ ext{job}} - u_{i,n,t}^{ ext{job}} - v_{i,n,t}^{ ext{sd}} + v_{i,n,t}^{ ext{su}} = 0, \ orall j \in \mathcal{J}, orall t \in \mathcal{T}, n \in \mathcal{N}, \end{aligned}$$

$$egin{aligned} v_{j,n,t}^{ ext{sd}} + v_{j,n,t}^{ ext{su}} \leq 1, \ orall j \in \mathcal{J}, t \in \mathcal{T}, n \in \mathcal{N}. \end{aligned}$$



	t = 1	L	OFF				
$u_{j,n,t}$	1	1	1	0	0		
v	$v^{su}=1$			$v^{sd} = 1$			

No start if the task is not assigned

Logical relationship between processing, start, and shutdown

No start and finish at same time





### 2. Current working jobs

$$egin{aligned} x_{j,N_j^w}^{ ext{job}} &= x_j^{ ext{Done}}, \ orall j \in \mathcal{J}^{ ext{w}}, \ u_{j,N_j^w,t}^{ ext{job}} &= x_j^{ ext{Done}}, \ orall j \in \mathcal{J}^{ ext{w}}, t \in [1,T_j^w]. \end{aligned}$$

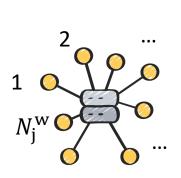






Final node state defines task completion

Completion must occur in time



	ON O		t=1	OFF)		
$u_{j,n,t}$	1	1	1	1	0	(DONE)
or						<b>DONE</b>
$u_{j,n,t}$	1	1	0	0	0	
						ı II





### 3. Time-sensitive job (in job pool)

$$egin{aligned} oldsymbol{u}_{j,n}^{ ext{job}} &= \sum_{t=1}^{T_j^{ ext{TS}}} oldsymbol{v}_{j,n,t}^{ ext{sd}}, \ orall j_{j,n,t}^{ ext{TS}} &= T_{j,n}^{ ext{job}} \cdot oldsymbol{x}_{j,n}^{ ext{job}}, \ orall j \in \mathcal{J}^{ ext{TS}}, n \in \mathcal{N}, \ &\sum_{t=1}^{T_{j,n}^{ ext{job}}} oldsymbol{v}_{j,n,t}^{ ext{su}} &= oldsymbol{u}_{j,n,t}^{ ext{TS}}, \ orall j \in \mathcal{J}^{ ext{TS}}, n \in \mathcal{N}, t \in [1, T_j^{ ext{TS}}], \ &\sum_{n \in \mathcal{N}} \sum_{t \in [1, T_j^{ ext{TS}}]} oldsymbol{v}_{j,n,t}^{ ext{sd}} &= oldsymbol{x}_j^{ ext{Done}}, \ orall j \in \mathcal{J}^{ ext{TS}}, \end{aligned}$$

Shut down mark defines training's completion;

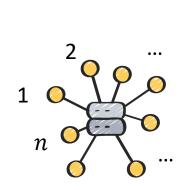
The task need to be training with task time;

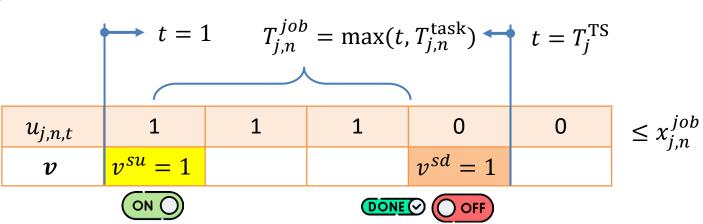
Once the task starts. It cannot be terminated;















ON ()

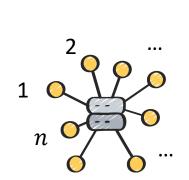
#### 4. Non-time-sensitive job (in job pool)

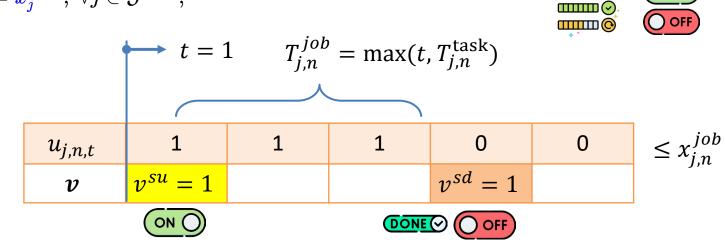
$$egin{aligned} u_{j,n}^{
m job} &= \sum_{t \in \mathcal{T}} v_{j,n,t}^{
m sd}, \ orall j \in \mathcal{J}^{
m NTS}, n \in \mathcal{N}, \ \sum_{t \in \mathcal{T}} u_{j,n,t}^{
m job} &= T_{j,n}^{
m job} \cdot x_{j,n}^{
m job}, \ orall j \in \mathcal{J}^{
m NTS}, n \in \mathcal{N}, \ \sum_{ au = 1}^{T_{j,n}^{
m job}} v_{j,n,(t- au+1)}^{
m su} &\leq u_{j,n,t}^{
m job}, \ orall j \in \mathcal{J}^{
m NTS}, n \in \mathcal{N}, t \in \mathcal{T}, \ \sum_{n \in \mathcal{N}} \sum_{t \in \mathcal{T}} v_{j,n,t}^{
m sd} &= x_{j}^{
m Done}, \ orall j \in \mathcal{J}^{
m NTS}, \end{aligned}$$

Shut down mark defines training's completion;

The task need to be training with task time;

Once the task starts. It cannot be terminated;









#### 5. Transferred job (1)

$$oldsymbol{x}_j^{ ext{TF}} = oldsymbol{x}_j^{ ext{Done}} - \sum_{\{n | \mathcal{I}^N(n) = \mathcal{I}^J(j)\}} oldsymbol{x}_{j,n}^{ ext{job}}, \ orall j \in \mathcal{J},$$

$$D_{j} \cdot x_{j}^{ ext{TF}} = \sum_{t=1}^{T_{ ext{range}}} y_{j,t}^{ ext{bw,out}}, \ orall j \in \mathcal{J}^{ ext{NTS}},$$

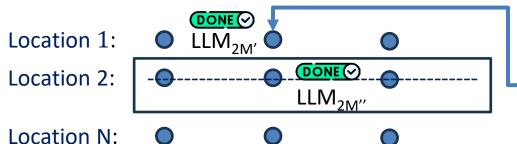
$$D_j \cdot oldsymbol{x}_j^{ ext{TF}} = \sum
olimits_{t=1}^{T_j^{ ext{TS}}} oldsymbol{y}_{j,t}^{ ext{bw,out}}, \ orall j \in \mathcal{J}^{ ext{TS}},$$

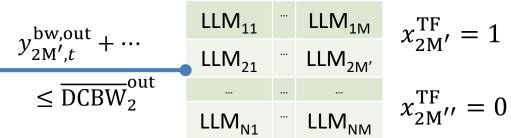
$$\sum_{\{j | \mathcal{I}^J(j) = i\}} oldsymbol{y_{j,t}^{ ext{bw,out}}} \leq \overline{ ext{DCBW}}_i^{ ext{out}}, \ orall i \in \mathcal{I}, \ orall t \in \mathcal{T},$$

Defines what task is transferred.

Once the task is transferred. The training data need to be upload/download to another location.

The uploading data size upper bound.





$$x_{2M''}^{TF} = 0$$





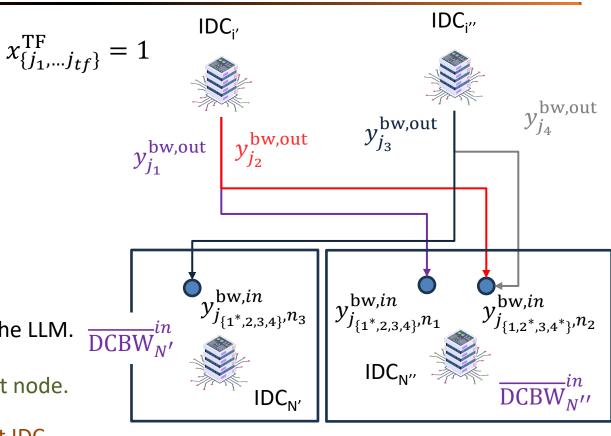
#### 5. Transferred job (2)

$$egin{aligned} oldsymbol{y_{j,t}^{ ext{bw,out}}} &= \sum_{\left\{n | \mathcal{I}^N(n) 
eq \mathcal{I}^J(j)
ight\}} oldsymbol{y_{j,n,t}^{ ext{bw,in}}}, \ orall j \in \mathcal{J}, t \in \mathcal{T}, \ oldsymbol{y_{j,n,t}^{ ext{bw,in}}} &\leq oldsymbol{ar{y}_{j,n,t}^{ ext{bw,in}}}, \ orall j \in \mathcal{J}, n \in \mathcal{N}, t \in \mathcal{T}, \ oldsymbol{ar{y}_{j,n,t}^{ ext{bw,in}}} &= oldsymbol{ar{y}^{ ext{bw,in}}} oldsymbol{x_{j,n}^{ ext{job}}}, \ orall j \in \mathcal{J}, n \in \mathcal{N}, t \in \mathcal{T}, \ \sum_{\left\{n | \mathcal{I}^N(n) = i
ight\}} oldsymbol{y_{j,n,t}^{ ext{bw,in}}} \leq oxdot{ ext{DCBW}}_i^{ ext{in}}, \ orall i \in \mathcal{I}, \ orall t \in \mathcal{T}, \end{aligned}$$

Select the receiving node\* to download the LLM.

The upper bound of downloading speed at node.

The upper bound of downloading speed at IDC.



$$x_{j_3,n_3}^{\text{job}} = x_{j_4,n_2}^{\text{job}} = x_{j_1,n_1}^{\text{job}} = x_{j_2,n_2}^{\text{job}} = 1$$





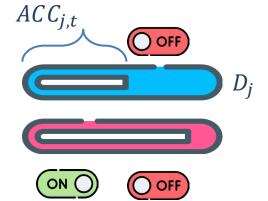
#### 5. Transferred job (3)

$$egin{aligned} ACC_{j,t} = \sum_{n \in \mathcal{N}} \sum_{ au=1}^{t} y_{j,n, au}^{ ext{bw,in}}, \ orall j \in \mathcal{J}, t \in \mathcal{T}, \end{aligned}$$

$$egin{aligned} ACC_{j,t} \geq D_{j,t}^{in}, \ orall j \in \mathcal{J}, t \in \mathcal{T}, \end{aligned}$$

$$m{D_{j,t}^{in}} = D_j \cdot m{v_{j,n,t+1}^{\mathrm{su}}}, \ orall j \in \mathcal{J}, n \in ig\{ n \ | \ \mathcal{I}^N(n) 
eq \mathcal{I}^J(j) ig\}, t \in \mathcal{T}.$$

The size of the data that has been downloaded. Training can only start after it is **completely downloaded**.



	$v_{j,n,t+1}$	Regulatio
$ACC_{j,t} \leq D_j$	{0}	
$ACC_{j,t} = D_j$	{0,1}	,

Job<sub>i</sub> can start now

Ensures LLM is only transmit to a single node

$$D_j \cdot oldsymbol{x_j^{ ext{TF}}} = \sum_{t=1}^{T_{ ext{range}}} oldsymbol{y_{j,t}^{ ext{bw,out}}}$$

$$oldsymbol{y_{j,t}^{ ext{bw,out}}} = \sum_{ig\{n|\mathcal{I}^N(n)
eq \mathcal{I}^J(j)ig\}} oldsymbol{y_{j,n,t}^{ ext{bw,in}}}$$





#### 6. Computing nodes Energy Modeling Overview







$$egin{aligned} e_{n,t}^{ ext{O,Node}} &= e_{n,t}^{ ext{O,N,idle}} + e_{n,t}^{ ext{O,N,w}}, \ orall n \in \mathcal{N}, t \in \mathcal{T}, \ e_{n,t}^{ ext{O,N,idle}} &= E_n^{ ext{O,N,idle}} u_{n,t}^{ ext{power}}, \ orall n \in \mathcal{N}, t \in \mathcal{T}, \ e_{n,t}^{ ext{O,N,w}} &= \sum_{j \in \mathcal{J}} E_{j,n}^{ ext{O,N,w}} \cdot eta_j^{ ext{TDP}} \cdot u_{j,n,t}^{ ext{job}}, \ orall n \in \mathcal{N}, t \in \mathcal{T}, \ u_{n,0}^{ ext{power}} &= 1, \ orall n \in \mathcal{N}^*, \ u_{j,n,t}^{ ext{job}} \leq u_{n,t}^{ ext{power}}, \ orall j \in \mathcal{J}, orall t \in \mathcal{T}, n \in \mathcal{N}, \ u_{n,t-1}^{ ext{power}} - u_{n,t}^{ ext{power,sd}} + v_{n,t}^{ ext{power,su}} &= 0, \ orall t \in \mathcal{T}, n \in \mathcal{N}. \end{aligned}$$



The power consumption of every node (idle, working)

The power state of every node (idle, working)

Node Type	Power (KW)	Performance (petaFLOPS)
10 DGX1	350	17.3
10 DGX2	100	20
4 DGXA100	26	20
10 DGXA100s	15	13
1 DGXA200	14.3	72
1 DGXH100	10.2	32

TABLE I: Power and Performance Specifications of Nodes





#### 7. HVAC, Temperature, and BESS System

Those constraints are referred to [1]. However, we made several changes based our setup.

#### 7.1 Modified Heating and Air Conditioning System

$$egin{aligned} T_{z,t}^{ ext{Zone}} &= T_{z,t-1}^{ ext{Zone}} + \sum_{z' \in adj(z)} \left(rac{T_{z',t-1}^{ ext{Zone}} - T_{z,t-1}^{ ext{Zone}}}{C_z^{ ext{heat}}R_{z'z}^{ ext{Zone}}}
ight) + rac{ heta_{z,t}}{C_z^{ ext{heat}}} \ &+ rac{\dot{m}_{z,t}^{ ext{Zone}}c^{ ext{a,s}}\left(T_{z,t}^{ ext{AC}} - T_{z,t-1}^{ ext{Zone}}
ight)}{C_z^{ ext{heat}}}, \ orall z \in \mathcal{Z}, orall t \in \mathcal{T}, \ & ext{where } C_z^{ ext{heat}} = c^{ ext{a,s}} \cdot 
ho^{ ext{air}} \cdot S_z^{ ext{Zone}} \cdot h_z, \ & ext{\dot{m}}_{z,t}^{ ext{Zone}} = k_z^{ ext{AC}} \cdot v_t^{ ext{AC}}, \end{aligned}$$

Time Discrete Difference Room
Temperature Model [2]

 $heta_{z,t} = \xi \sum_{m{E}_{m{n},t}^{ ext{O,Node}}} m{E}_{m{n},t}^{ ext{O,Node}}.$  Heat from the local computing nodes.

[1] Zhao, Zhongqi, Lei Fan, and Zhu Han. "Optimal Data Center Energy Management with Hybrid Quantum-Classical Multi-Cuts Benders' Decomposition Method." IEEE Transactions on Sustainable Energy (2023).

[2] Belić, Filip, Željko Hocenski, and Dražen Slišković. "Thermal modeling of buildings with RC method and parameter estimation." 2016 International Conference on Smart Systems and Technologies (SST). IEEE, 2016.





Algorithm: Hybrid Quantum-classical Nonlinear Benders' decomposition Approach

Step 1: Reformulate the objective function

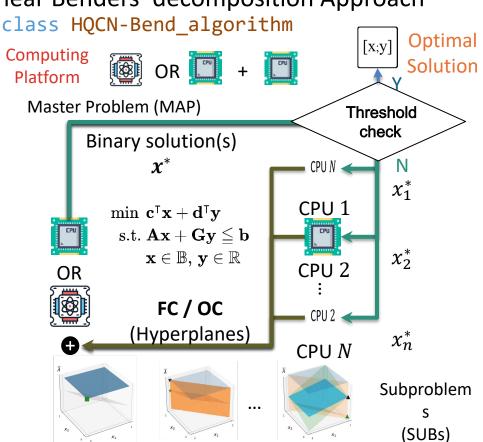
$$f(\mathbf{x}') = \mathbf{x}'^{\mathsf{T}} \mathbf{Q}_{\mathrm{QUBO}} \mathbf{x}'$$

$$\mathbf{x}^{\mathsf{T}}\operatorname{diag}(\mathbf{c})\mathbf{x}$$

$$+\sum_{i=-\underline{m}}^{\overline{m}_+}w_{i+\underline{m}}2^iw_{i+\underline{m}}-\sum_{j=0}^{\overline{m}_-}w_{j+(1+\underline{m}+\overline{m}_+)}2^jw_{j+(1+\underline{m}+\overline{m}_+)}$$

$$+\sum_{k\in K}P_k\Bigg(\overline{\lambda}(\mathbf{w})+ig(u^kig)^{\intercal}\mathbf{A}\mathbf{x}+\sum_{l=0}^{\overline{l}^K}2^ls_{kl}^K-b^{\intercal}u^k\Bigg)^2$$

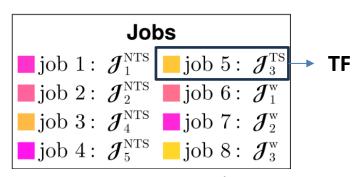
$$+\sum_{j\in J}P_j\Bigg(ig(r^jig)^{\intercal}\mathbf{A}\mathbf{x}+\sum_{l=0}^{ar{l}^J}2^ls_{kl}^J-\mathbf{b}^{\intercal}r^j\Bigg)^2.$$



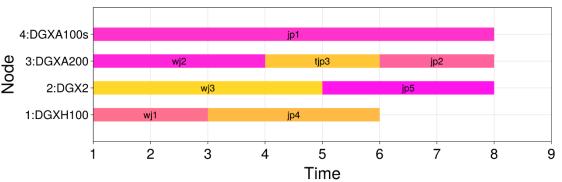




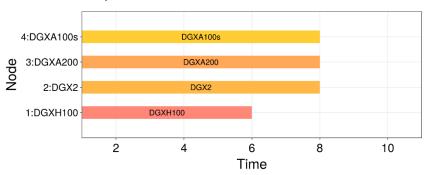
☐ Simulation Results:

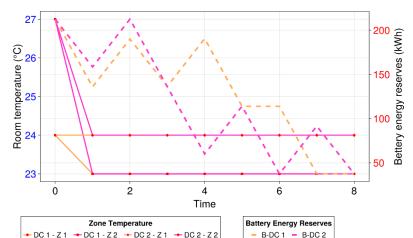


LLM Training Task scheduling



- The LLM training task / device arrangement is valid
- Zone temperature is within the bound





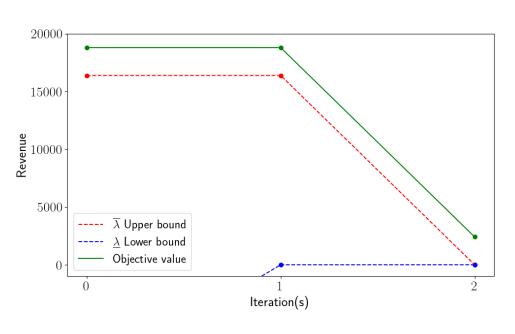




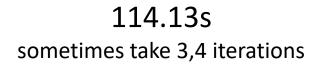
#### ☐ Simulation Results:

• Benders Decomposition Performance and Convergence:

Solver access time / iter.









The GPU runs faster than the CPU based algorithm

Goto, H., Tatsumura, K., & Dixon, A. R. (2019). Combinatorial optimization by simulating adiabatic bifurcations in nonlinear Hamiltonian systems. Science advances, 5(4), eaav2372.

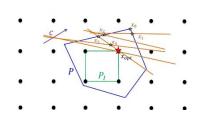
https://www.openjij.org/

### **Future Work & Conclusion**

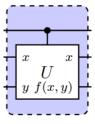




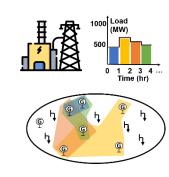
- ☐ Future Work
  - HQC-Bend package Upgrade in for MICP
    - Benders' Dual / General BD / Logic BD
    - High-order unconstraint binary optimization (HUBO) in Obj./Constraint
      - $\prod x_i \cdot x_j \cdot ... \cdot x_k$  to Digital Q Circuit / QUBO (QA)

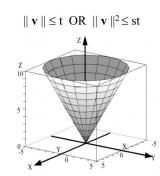


**Plane-cutting Method** 



- Internet Data Center Model Upgrade
  - Unit commitment convex constraint plug-in (MISOCP)
  - Dynamic flexible training LLM task scheduling
  - Quantum communications





**Internet Data Center Model** 

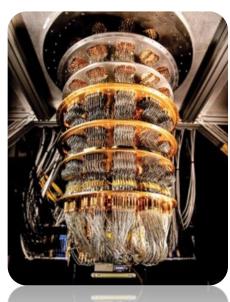
## **Future Work & Conclusion**





#### Conclusion

- Quantum Computing (QC) provide a special way to deal with the complex MIP. By leverage Both QC and classical computation power, HQC-Bend can reduce the computation time for Mixed-integer Programming significantly.
- Work 1: HQC-Bend for MILP and Python Package
  - ✓ Reformulate the MAP of BD for the MILP problem and validate the algorithm.
  - ✓ Introduce a Python package implementing the HQC-Bend algorithm.
- Work 2: Energy Management Problem in Internet Data Center Using HQC-Bend
  - ✓ Propose a MILP model for IDC energy management.
  - ✓ the HQC-Bend approach outperforms the CBD approach in practice.
- Work 3: Optimal Energy Management and LLM Training Job Scheduling for IDC Using Nonlinear HQC-Bend
  - ✓ Propose a MINLP model for IDC LLM task scheduling & energy management
  - ✓ The HQCN-Bend method is feasible for solving certain MINLPs.





### **Publications**





#### Journal

- 1. Zhao, Z., Fan, L., & Han, Z. (2023). Optimal Data Center Energy Management with Hybrid Quantum-Classical Multi-Cuts Benders' Decomposition Method. IEEE Transactions on Sustainable Energy.
- 2. Xuan, W., Zhao, Z., Fan, L., & Han, Z. (2024). Lagrangian Relaxation Based Parallelized Quantum Annealing and its Application in Network Function Virtualization. IEEE Open Journal of the Communications Society.

#### Conference

- 1. Zhao, Z., Fan, L., & Han, Z. (2022, April). Hybrid quantum benders' decomposition for mixed-integer linear programming. In 2022 IEEE Wireless Communications and Networking Conference (WCNC) (pp. 2536-2540). IEEE.
- 2. Xuan, W., Zhao, Z., Fan, L., & Han, Z. (2021, October). Minimizing delay in network function visualization with quantum computing. In 2021 IEEE 18th International Conference on Mobile Ad Hoc and Smart Systems (MASS) (pp. 108-116). IEEE.
- 3. Zhao, Z., Fan, L., Guo, Y., Wang, Y., Han, Z., & Hanzo, L. (2024, June). QAOA-assisted benders' decomposition for mixed-integer linear programming. In ICC 2024-IEEE International Conference on Communications (pp. 1127-1132). IEEE.
- 4. Zhao, Z., Fan, L., Zheng, H., & Han, Z. (2023, October). Quantum Computing for Cable-Routing Problem in Solar Power Plants. In 2023 North American Power Symposium (NAPS) (pp. 1-6). IEEE.
- 5. Zhao, Z., Fan, L., & Han, Z. (2024, October). Optimal Energy and IT Service Emergency Schedule for Internet Data Center. In 2024 56th North American Power Symposium (NAPS) (pp. 1-6). IEEE.
- 6. Zhao, Z., Yao, Y., Fan, L., & Ding, F. (2024, July). Spatial-Temporal PV Hosting Capacity Estimation and Evaluation. In 2024 IEEE Power & Energy Society General Meeting (PESGM) (pp. 1-5). IEEE.

#### Conference/Demo

- 1. Zhao, Z, Mingze Li, Lei Fan, and Zhu Han. "HQC-Bend: A Python Package of Hybrid Quantum-Classical Multi-cuts Benders' Decomposition Algorithm." 2025 IEEE International Conference on Quantum Communications, Networking, and Computing (QCNC) / Computer Communication (INFOCOM) / International Conference on Communications (ICC). IEEE, 2025.
- 2. Zhao, Z, Lei Fan, and Zhu Han. "Optimal Data Center Energy Management and LLM Task Scheduling with Hybrid Quantum-Classical Nonlinear Benders' Decomposition Method." 2025, Ongoing.

## Should I approve Zhongqi's Defense?

**Approve** 



Still

**Approve** 



# Thank you!

## Best Professors Ever!

















